

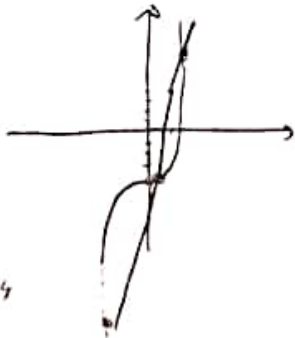
اساسيات يا نظم: A

يا سخفا نه بگفتن شما ره: 11

14, 15

باسمه تعالی
نام و نام خانوادگی: علی رابین

$$f(x) = \begin{cases} x^2 - 4 & : x > 5 \\ 12x - 2 & : x \leq 5 \end{cases}$$



$$\begin{aligned} & x^2 - 12x + 4 \geq 0 \\ & x^2 - 12x + 4 = 0 \\ & x = \frac{12 \pm \sqrt{144 - 16}}{2} = \frac{12 \pm \sqrt{128}}{2} = \frac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2} \end{aligned}$$

$$12x - 2 \leq x^2 - 4$$

$$12x - 2 \leq x^2 - 4 \Rightarrow 0 \leq x^2 - 12x + 4$$

$$\frac{-x}{-6} + \frac{x}{6} +$$

$$\Rightarrow x = [-4, +\infty)$$

2

$$f(x) = 2x + k \quad f^{-1}(x) = x \Rightarrow f(x) = x$$

$$f(x) = 2x + k \Rightarrow 12x + k = x \Rightarrow k = -11$$

$$f(7) = 2 \times 7 - 11 = 11$$

$$f(f(x)) = 2(2x - 11) - 11 = 4x - 33$$

3

$$f: A \rightarrow B \quad f(x) = \frac{x^2}{x-1} \Rightarrow y = \frac{x^2}{x-1}$$

$$f = (5, 25) \Rightarrow f(x) = \frac{x^2}{x-1} = 25$$

$$x^2 = 25(x-1) \Rightarrow x^2 - 25x + 25 = 0 \Rightarrow x = \frac{25 \pm \sqrt{625 - 100}}{2} = \frac{25 \pm \sqrt{525}}{2}$$

4

$$f = \{(5, 5), (2, 7), (7, 2), (9, 4)\} \quad g = \{(5, 2), (8, 6), (9, 8)\}$$

$$f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (4, 4)\}$$

$$f^{-1} = \{(5, 2), (7, 4), (2, 7), (4, 9)\}$$

$$f^{-1} \circ f = \{(2, 2), (4, 4), (7, 7), (9, 9)\}$$

$$g^{-1} = \{(2, 5), (6, 8), (8, 9)\}$$

$$f \circ g^{-1} = \{(2, 5), (5, 6)\}$$

$$g \circ f = \{(5, 9), (7, 2), (9, 8)\}$$

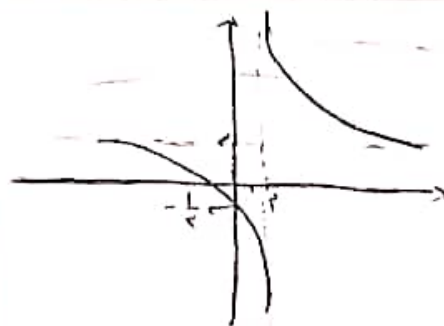
5

$$f = \{(2, 4), (4, 8), (6, 0)\} \quad g = \{(2, 1), (4, 2), (6, 3)\} \quad h = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$$

$$g^{-1} = \{(1, 2), (2, 4), (3, 6)\}$$

$$\frac{h}{f \circ g^{-1}} = \frac{\{(1, 2), (2, 4), (3, 6), (4, 8)\}}{\{(2, 4), (4, 8), (6, 0)\}} = \{(1, 2), (2, 4)\} = \{(1, \frac{1}{2}), (2, \frac{1}{4})\}$$

6



$$y = \frac{x^2 + 1}{x - 2}$$

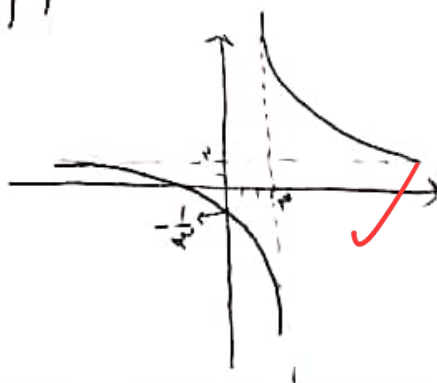
$$y = x + 5$$

$$y = x + 5$$

- 9

$$y = \frac{x^2 + 1}{x - 2} \Rightarrow x^2 + 1 = (x - 2)(x + 5)$$

$$(x - 2)x = -x - 1$$



$$x = \frac{y + 1}{y - 2}$$

$$x = \frac{-y - 1}{y - 2}$$

$$y = x + 5$$

$$y = x + 5$$

$$y = \frac{x^2 + 1}{x - 2}$$

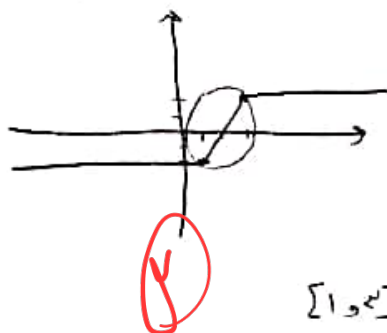
$$y = \frac{x^2 + 1}{x - 2}$$

$$f(x) = |x - 1| - |x - 2| \Rightarrow$$

$$[a, b] = [1, 2]$$

- 10

$$R = [-1, 1]$$



$$x - 1 + x - 2 = 2x - 3$$

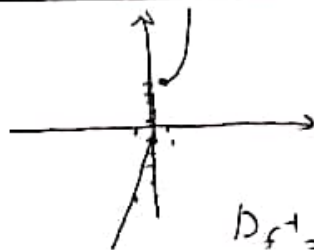
$$y = 2x - 3 \Rightarrow x = \frac{y + 3}{2}$$

$$y = \frac{x + 3}{2}$$

$$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ 2x - 1 & x < 1 \end{cases}$$

$$D_f = (-\infty, 1] \cup [1, +\infty)$$

$$R_f = (-\infty, 1] \cup [1, +\infty)$$



$$f^{-1}(x) = \begin{cases} \sqrt{x - 4} & x \geq 5 \\ \frac{x + 1}{2} & x < 5 \end{cases}$$

$$D_{f^{-1}} = (-\infty, 1] \cup [1, +\infty)$$

$$R_{f^{-1}} = (-\infty, 1] \cup [1, +\infty)$$

$$y = x^2 + 4 \Rightarrow x^2 = y - 4 \Rightarrow x = \sqrt{y - 4}$$

$$y = 2x - 1 \Rightarrow 2x = y + 1 \Rightarrow x = \frac{y + 1}{2}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d}$$

$$(b < 0)$$

$$\Delta$$

- 11

$$\frac{x^2 + 1}{x + 2} = \frac{(x+1)^2}{x+2} \Rightarrow x^2 + 1 = \frac{(x+1)^2}{x+2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} = \frac{-x+3}{x+2}$$

$$f^{-1}(b) = \frac{-b+3}{b+2} = \frac{1}{2} = \Delta$$

$$f^{-1}(x) = y = \frac{x+1}{-x-2}$$

$$-x - 1 = y(x + 2)$$

$$(-x - 1) = y(x + 2)$$

$$(-x - 1) = y(x + 2)$$

$$f(z) = \frac{z}{z^2 + 1}$$

$$yz^2 + y = z$$

$$yz^2 + y - z = 0$$

$$\Delta = 1 - 4y^2$$

$$z = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

-10

$$f(1) = \frac{1}{2} \Rightarrow f^{-1}\left(\frac{1}{2}\right) = 1$$

$$y = \frac{1 \pm \sqrt{1 - 4z^2}}{2z} \Rightarrow 1 = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

~~1, 1/2~~

$$y = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

$$f(z) = \frac{z}{\Delta} \Rightarrow f^{-1}\left(\frac{z}{\Delta}\right) = z$$

$$z = \frac{1 \pm \sqrt{1 - \frac{17}{\Delta^2}}}{\frac{z}{\Delta}} \sim \frac{1}{\Delta} = \frac{z}{\Delta}$$

$$\frac{\Delta}{\frac{z}{\Delta}} = \frac{\Delta}{\Delta} \times \frac{\Delta}{z} = \Delta \checkmark \quad \frac{\Delta}{\frac{z}{\Delta}} = \frac{\Delta}{\Delta} \times \frac{\Delta}{z} = \frac{1}{z} \times \Rightarrow f^{-1}(z) = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

$$|z| \leq \frac{1}{4}$$