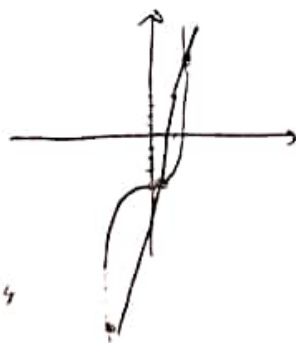


$$f(x) = \begin{cases} x^2 - 4 & : x > 5 \\ 12x - 2 & : x \leq 5 \end{cases}$$



$$\begin{aligned} & x^2 - 12x + 4 + 12x + 4 \\ & \frac{x^2 - 4x^2}{-4x^2 - 12x + 16} \\ & \frac{+4x^2 + 12x}{4x + 16} \end{aligned}$$

$$\frac{x^2 - 4x^2 + 4}{(x-2)^2}$$

$$12x - 2 \leq x^2 - 4$$

$$12x - 2 \leq x^2 - 4 \Rightarrow 0 \leq x^2 - 12x + 4$$

$$\frac{-x}{-6} + \frac{x}{6} +$$

$$\Rightarrow x = [-4, +\infty)$$

$$f(x) = x^2 + k \quad f^{-1}(x) = x \Rightarrow f(x) = x$$

$$f(x) = x^2 + k \Rightarrow 12x - 2 = x^2 \Rightarrow k = -1$$

$$الف) f(7) = 2 \times 7 - 1 = 11$$

$$ب) f(f(x)) = x^2(x^2 - 1) + 1 = 9k - 4$$

۳- ۲

$$f: A \rightarrow B \quad f(x) = \frac{x^2}{x-1} \Rightarrow y = \frac{x^2}{x-1}$$

$$f = (5, 25) \Rightarrow f(x) = \frac{x^2}{x-1} = 25$$

$$x^2 = 25x^2 - 25x$$

$$0 = x^2 - 25x \Rightarrow x = 0, 25$$

چون تابع ثابت $f(x) = 0$ موجود است

عق

۴- ۳

$$f = \{(5, 5), (4, 7), (7, 2), (9, 4)\} \quad g = \{(5, 2), (8, 6), (9, 8)\}$$

$$الف) f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (4, 4)\}$$

$$f^{-1} = \{(5, 2), (7, 4), (2, 7), (4, 9)\}$$

$$ب) f^{-1} \circ f = \{(2, 5), (4, 7), (7, 2), (9, 4)\}$$

$$g^{-1} = \{(2, 5), (6, 8), (8, 9)\}$$

$$ج) f \circ g^{-1} = \{(2, 5), (5, 6)\}$$

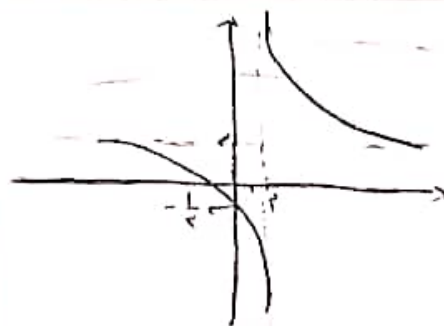
$$د) g \circ f = \{(5, 9), (7, 2), (9, 8)\}$$

۵- ۴

$$f = \{(5, 4), (4, 8), (6, 0)\} \quad g = \{(5, 1), (4, 2), (6, 5)\} \quad h = \{(1, 2), (3, 4), (4, 1), (5, 1)\}$$

$$g^{-1} = \{(1, 5), (2, 4), (5, 6)\}$$

$$\frac{h}{f \circ g^{-1}} = \frac{\{(1, 2), (3, 4), (4, 1), (5, 1)\}}{\{(1, 4), (2, 8), (5, 0)\}} = \left\{ \left(1, \frac{2}{4}\right), \left(3, \frac{4}{8}\right) \right\} = \left\{ \left(1, \frac{1}{2}\right), \left(3, \frac{1}{2}\right) \right\}$$



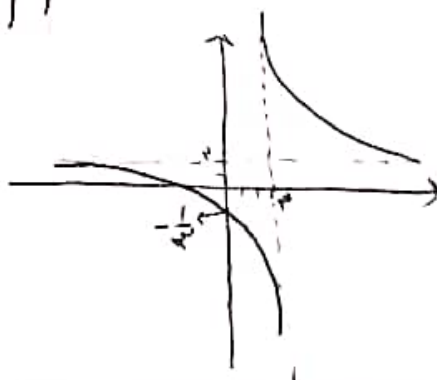
$$y = \frac{x^2 + 1}{x - 2}$$

$$y = 2 \Rightarrow \text{مجاوب قائم}$$

$$y = 2 \Rightarrow \text{مجاوب افق}$$

- 9

$$y = \frac{x^2 + 1}{x - 2} \Rightarrow y(x - 2) = x^2 + 1 \Rightarrow x^2 - yx + 2y - 1 = 0$$



$$x = \frac{y + 1}{y - 2} \Rightarrow y = \frac{x^2 + 1}{x - 2}$$

$$y = 2 \Rightarrow \text{مجاوب قائم}$$

$$y = 2 \Rightarrow \text{مجاوب افق}$$

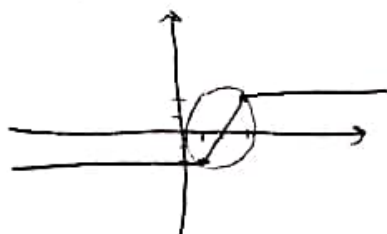
$$y = \frac{x^2 + 1}{x - 2}$$

$$f(x) = |x - 1| - |x - 2| \Rightarrow \text{تابع سراسر}$$

$$[a, b] = [1, 2]$$

- 7

$$R = [-1, 1]$$



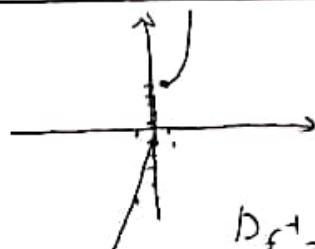
$$x - 1 + x - 2 = 2x - 3 \Rightarrow y = 2x - 3 \Rightarrow x = \frac{y + 3}{2}$$

$$y = \frac{x + 3}{2} \Rightarrow x = 2y - 3$$

$$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ 2x - 1 & x \leq 0 \end{cases}$$

$$D_f = (-\infty, 0] \cup [1, +\infty)$$

$$R_f = (-\infty, -1] \cup [5, +\infty)$$



$$f^{-1}(x) = \begin{cases} \sqrt{x - 4} & x \geq 5 \\ \frac{x + 1}{2} & x \leq -1 \end{cases}$$

$$f^{-1}(x) =$$

$$D_{f^{-1}} = (-\infty, -1] \cup [5, +\infty)$$

$$R_{f^{-1}} = (-\infty, -1] \cup [5, +\infty)$$

$$y = x^2 + 4 \Rightarrow x^2 = y - 4 \Rightarrow x = \sqrt{y - 4}$$

$$y = 2x - 1 \Rightarrow 2x = y + 1 \Rightarrow x = \frac{y + 1}{2}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d}$$

$$(b < 0)$$

$$\Delta$$

- 9

↓

$$\frac{x^2 - \frac{(x+1)^2}{x+2}}{x+2} = \frac{x^2 - \frac{x^2 + 2x + 1}{x+2}}{x+2} = \frac{-\frac{(x+1)}{x+2}}{x+2} = -\frac{(x+1)}{(x+2)^2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} = \frac{-\frac{1}{2}}{x+2} \Rightarrow b = -\frac{1}{2}$$

$$f^{-1}(b) = \frac{-\frac{1}{2} - \frac{1}{2}}{-\frac{1}{2} + 2} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$$

$$f^{-1}(x) = y = \frac{x+1}{-x-2}$$

$$-x - 1 = yx + 2y$$

$$(-x - y)x = 2y + 1$$

$$\Rightarrow x = \frac{2y + 1}{-x - y}$$

$$f(z) = \frac{z}{z^2 + 1}$$

$$yz^2 + y = z$$

$$yz^2 + y - z = 0$$

$$\Delta = 1 - 4y^2$$

$$z = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

-10

$$f(1) = \frac{1}{2} \Rightarrow f^{-1}\left(\frac{1}{2}\right) = 1$$

$$y = \frac{1 \pm \sqrt{1 - 4z^2}}{2z} \Rightarrow 1 = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

$$y = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$

$$f(z) = \frac{z}{\Delta} \Rightarrow f^{-1}\left(\frac{z}{\Delta}\right) = z$$

$$z = \frac{1 \pm \sqrt{1 - \frac{1z}{\Delta}}}{\frac{z}{\Delta}} \sim \frac{1}{\frac{z}{\Delta}} = \frac{\Delta}{z}$$

$$\frac{\frac{\Delta}{z}}{\frac{z}{\Delta}} = \frac{\Delta}{z} \times \frac{\Delta}{z} = \Delta \checkmark \quad \frac{\frac{z}{\Delta}}{\frac{z}{\Delta}} = \frac{z}{\Delta} \times \frac{\Delta}{z} = 1 \times \Rightarrow f^{-1}(z) = \frac{1 \pm \sqrt{1 - 4z^2}}{2z}$$