

$f(n) = \begin{cases} n^3 - 4 & ; n > a \\ 12n - 2 & ; n \leq a \end{cases}$

$n^3 - 4 \gg 12n - 2 \rightarrow n^3 - 12n + 16 \gg 0 \rightarrow (n-2)^2(n+4) \leq 0$

ریشه های حارمه: مضاعف ۲، ۴ و ۶ $(n-2)^2(n+4)$

در واقع با دستفان زدن و تبیین علت و کم کردن کردن
 روش اول باید مستحب باشد.

$\frac{-4}{-4+4+4} \rightarrow a = [-4, +\infty)$

(۲)

$f(n) = 3n + k \xrightarrow{f^{-1}(2)=4} f(4) = 3(4) + k = 2 \rightarrow k = -10$

$\rightarrow f(n) = 3n - 1$

$f(7) = 21 - 1 = 20$

$f(f(n)) = 3(3n - 1) - 1 = 9n - 3 - 1 = 9n - 4$

(الف) (۲)

$y = \frac{an}{n-1} \xrightarrow{(a \neq 1)} ya = \frac{a^2}{a-1} \rightarrow ya^2 - ya = a^2 \rightarrow a^2 - ya = 0$

$\rightarrow a = 200 \rightarrow$ منفرجه قبل

$\Rightarrow a = 21$

(۲)

$f^{-1}(n) = \{(5, 2), (7, 4), (9, 6), (11, 8)\}$

$g^{-1}(n) = \{(3, 2), (4, 3), (5, 4), (6, 5)\}$

$f \circ f^{-1} = \{(5, 5), (7, 7), (9, 9), (11, 11)\}$

$f^{-1} \circ f = \{(2, 2), (4, 4), (6, 6), (8, 8)\}$

$f \circ g^{-1} = \{(2, 5), (3, 7), (4, 9), (5, 11)\}$

$g^{-1} \circ f = \{(3, 2), (4, 3), (5, 4), (6, 5)\}$

(الف) (ب) (ج) (د) (۲)

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{2}{4}), (3, \frac{4}{1}), (4, \frac{1}{2}), (9, \frac{1}{6})\}$

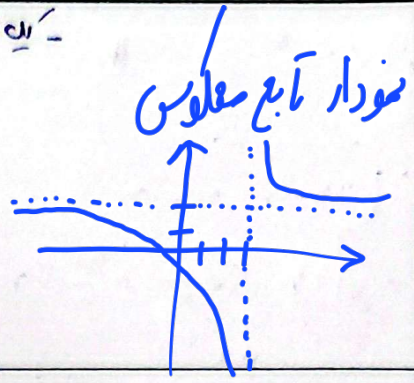
$\Rightarrow \frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{2})\}$

(۲)

$y = \frac{2x+1}{x-2} \rightarrow$ \rightarrow نمودار تابع معلوم است و خطوط نژاد مشخص است.

$\Rightarrow x = \frac{y+1}{y-2} \Rightarrow y = \frac{2x+1}{x-2} \rightarrow y = \frac{2x+1}{x-2}$

(1, 0) ✓



$f(x) = |x-1| - |x-3| \rightarrow$ \rightarrow $R = [-2, 2]$ \rightarrow $C_f = [1, 3]$ \rightarrow خطوط نژاد مشخص است ✓

\Rightarrow $y = 2x - 1 \rightarrow x = \frac{y+1}{2} \rightarrow y = \frac{x+1}{2} ; D_f = [2, 2]$

(2) ✓

$f(x) = \begin{cases} x^2 + 4 & ; x > 1 \\ \epsilon x - 1 & ; x \leq 1 \end{cases} \Rightarrow$

$x^2 + 4 \xrightarrow{x > 1} R_f = [2, +\infty)$
 $\epsilon x - 1 \xrightarrow{x \leq 1} R_f = (-\infty, -1]$

$y = x^2 + 4 \rightarrow x = \sqrt{y-4} \rightarrow y = \sqrt{x-4}$
 $y = \epsilon x - 1 \rightarrow x = \frac{y+1}{\epsilon} \rightarrow y = \frac{x+1}{\epsilon}$

$\Rightarrow f^{-1}(x) = \begin{cases} \sqrt{x-4} & ; x \in [2, +\infty) \\ \frac{x+1}{\epsilon} & ; x \in (-\infty, -1] \end{cases}$

(2) ✓

$y = x^2 \frac{(x+1)^2}{x+3} \rightarrow x = y^{\frac{1}{2}} \frac{(y^{\frac{1}{2}}+1)^2}{y^{\frac{1}{2}}+3} \rightarrow x = \frac{y^{\frac{1}{2}} + 3y^{\frac{1}{2}} - (y^{\frac{1}{2}}+1 + 3y^{\frac{1}{2}}+3y)}{y^{\frac{1}{2}}+3} = \frac{-y-1}{y^{\frac{1}{2}}+3}$

$\rightarrow y = -\frac{2x+1}{x+3} = \frac{-2x-1}{x+3} = \frac{ax+b}{cx+d} \rightarrow a = -2, b = -1, c = 1, d = 3$

$\rightarrow y = \frac{-9x-2}{2x+7}$

$f^{-1}(b) = f^{-1}(-2) = \frac{-9(-2)-2}{2(-2)+7} = \frac{18-2}{-4+7} = \frac{16}{3}$

(2) ✓

$y = \frac{x}{x^2+1} \rightarrow x = \frac{y}{y^2+1} \rightarrow x = xy^2 - y + x = 0$

$\rightarrow y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4x^2}}{2x}$ $\rightarrow y = \frac{1 + \sqrt{1 - 4x^2}}{2x}$ $|x| < \frac{1}{2}$

(1, 0) ✓