

$$f_{(x)} = \begin{cases} x^3 - \varepsilon & x > a \\ 12x - 2 & x \leq a \end{cases} \xrightarrow{\text{در } x=a} x^3 - \varepsilon \geq 12x - 2$$

$$\rightarrow x^3 - 12x + 14 > 0 \quad \frac{-\varepsilon}{-4} \quad \frac{2}{+4} \quad +$$

$$\rightarrow x \in [-\varepsilon, +\infty) \rightarrow a \in [-\varepsilon, +\infty) \quad \checkmark \quad \textcircled{2}$$

$$f_{(x)} = 3x + k \quad f_{(1)}^{-1} = \varepsilon \xrightarrow{x=\varepsilon} f_{(x)} = 12 + k = 2 \rightarrow k = -10$$

$$\rightarrow f_{(x)} = 3x - 10$$

الف) $f_{(v)} = (3 \times v) - 10 = 11 \quad \checkmark$ $\textcircled{2}$

ب) $f(f_{(x)}) = 3(3x - 10) - 10 = 9x - 40 \quad \checkmark$

$$f_{(x)} = \frac{ax}{x-1} \quad f_{(ra)}^{-1} = a \xrightarrow{x=a} \frac{a^2}{a-1} = 2a \rightarrow a^2 = 2a^2 - 2a \rightarrow a^2 = 2a$$

$$\rightarrow \textcircled{a=2} \quad \checkmark \quad \textcircled{2}$$

$$\rightarrow f_{(x)} = \frac{2x}{x-1} \quad f_{(x)}^{-1} = \frac{x}{x-2} \quad A(\varepsilon, 2)$$

$$f = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad f^{-1} = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad g = \{(2, 2), (2, 3), (3, 2)\} \quad g^{-1} = \{(2, 2), (2, 3), (3, 2)\}$$

الف) $f \circ f^{-1} = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad \checkmark$ ب) $f \circ g^{-1} = \{(2, 2), (2, 3)\} \quad \checkmark$

ج) $f^{-1} \circ f = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad \checkmark$ د) $g^{-1} \circ f = \{(2, 2), (2, 3), (3, 2)\} \quad \checkmark$

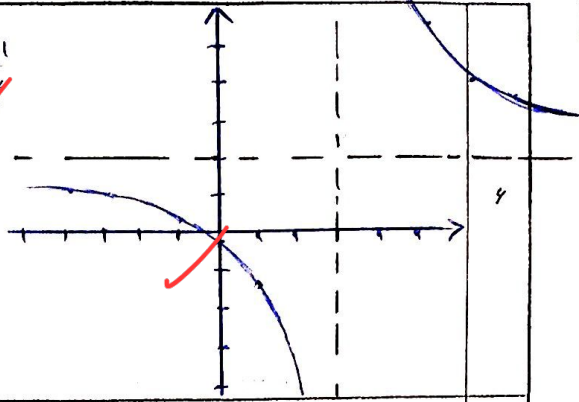
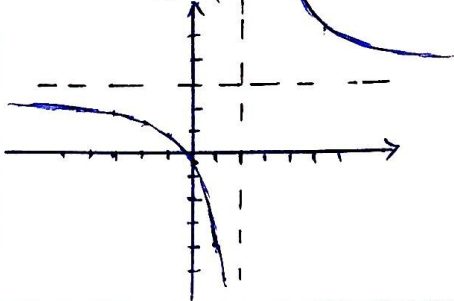
$$f = \{(2, 2), (2, 3), (3, 2)\} \quad h = \{(1, 2), (2, 3), (3, 1), (2, 1)\} \quad g = \{(2, 1), (2, 3), (3, 2)\}$$

$$g^{-1} = \{(2, 2), (2, 3), (3, 2)\} \rightarrow f \circ g^{-1} = \{(1, 2), (2, 3), (2, 1)\}$$

$$\rightarrow \frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{1}{2}\right), \left(2, \frac{1}{3}\right) \right\} \quad \checkmark \quad \textcircled{2}$$

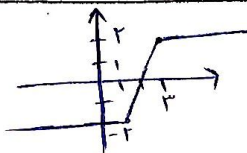
$$f(x) = \frac{x+1}{x-2}$$

$$x = \frac{y+1}{y-2} \rightarrow f^{-1} = \frac{y+1}{y-2}$$



(2)

$$f(x) = |x-1| - |x-2|$$



در بازه [1, 3] دارون پیوسته است

$$x \in [1, 3] \rightarrow y = |x-1| - |x-2| = x-1 - (2-x) = 2x-2 \rightarrow x = \frac{y+2}{2} \rightarrow f^{-1} = \frac{y+2}{2}$$

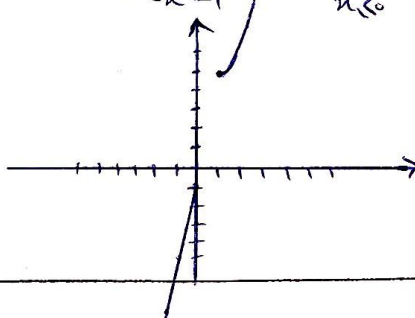
(1.5)

-2 < y < 2

$$f(x) = \begin{cases} x^3 + \varepsilon & x \geq 1 \\ \varepsilon x - 1 & x \leq 0 \end{cases}$$

$$y = x^3 + \varepsilon \rightarrow x^3 = y - \varepsilon \rightarrow x = \sqrt[3]{y - \varepsilon}$$

$$y = \varepsilon x - 1 \rightarrow \varepsilon x = y + 1 \rightarrow x = \frac{y+1}{\varepsilon}$$



$$f^{-1} = \begin{cases} \sqrt[3]{y - \varepsilon} & y \geq \varepsilon \\ \frac{y+1}{\varepsilon} & y \leq -1 \end{cases}$$

(2)

$$f(x) = x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2(x+2) - (x^2 + 2x + 1)}{x+2} = \frac{-x-1}{x+2}$$

$$\rightarrow x = \frac{-y-1}{y+2} \rightarrow f^{-1} = \frac{-x-1}{x+2} = \frac{-y-1}{y+2} = \frac{ax+b}{cx+d} \rightarrow \begin{cases} a=-1 \\ b=-1 \\ c=1 \\ d=2 \end{cases}$$

$$\rightarrow f^{-1}(\frac{1}{2}) = \frac{1}{2} = \frac{1}{2}$$

$$f(x) = \frac{x}{x^2+1}$$

(1.5)

این تابع یک به یک نیست به تابع دارون ندارد

$$f(x) = \frac{x}{5}$$

به شکل دایره

$$f(\frac{1}{2}) = \frac{1}{2} = \frac{\varepsilon}{1} = \frac{1}{2}$$

$$y = \frac{x}{x^2+1} \rightarrow yx^2 - x + y = 0 \rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

1/2 < y < 1/2

به چپ و راست هم علامت بین فاصله قابل قبول