

$$f_{(x)} = \begin{cases} x^3 - \varepsilon & x > a \\ 12x - 2 & x \leq a \end{cases} \xrightarrow{\text{در } x=a} x^3 - \varepsilon \gg 12x - 2$$

$$\rightarrow x^3 - 12x + 14 > 0 \quad \frac{-\varepsilon}{-4} + \frac{2}{4} +$$

$$\rightarrow x \in [-\varepsilon, +\infty) \rightarrow a \in [-\varepsilon, +\infty)$$

$$f_{(x)} = 3x + k \quad f_{(1)}^{-1} = \varepsilon \xrightarrow{x=\varepsilon} f_{(x)} = 12 + k = 2 \rightarrow k = -10$$

$$\rightarrow f_{(x)} = 3x - 10$$

$$\text{الف)} f_{(v)} = (3xv) - 10 = 11$$

$$\text{ب)} f(f_{(x)}) = 3(3x - 10) - 10 = 9x - 40$$

$$f_{(x)} = \frac{ax}{x-1} \quad f_{(ra)}^{-1} = a \xrightarrow{x=a} \frac{a^2}{a-1} = 2a \rightarrow a^2 = 2a^2 - 2a \rightarrow a^2 = 2a$$

$$\rightarrow \boxed{a=2}$$

$$\rightarrow f_{(x)} = \frac{2x}{x-1} \quad f_{(x)}^{-1} = \frac{x}{x-2} \quad A(\varepsilon, 2)$$

$$f = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad f^{-1} = \{(2, 2), (2, 3), (3, 2), (4, 4)\} \quad g = \{(3, 2), (4, 1), (4, 2)\} \quad g^{-1} = \{(2, 3), (1, 4), (2, 4)\}$$

$$\text{الف)} f \circ f^{-1} = \{(2, 2), (2, 3), (3, 2), (4, 4)\}$$

$$\text{ج)} f \circ g^{-1} = \{(2, 2), (2, 3)\}$$

$$\text{ب)} f^{-1} \circ f = \{(2, 2), (2, 3), (3, 2), (4, 4)\}$$

$$\text{د)} g^{-1} \circ f = \{(3, 2), (4, 1), (4, 2)\}$$

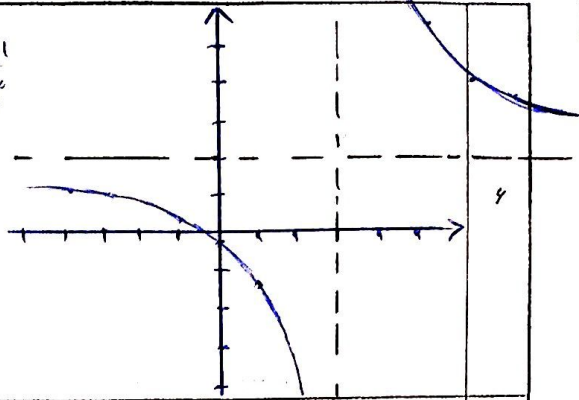
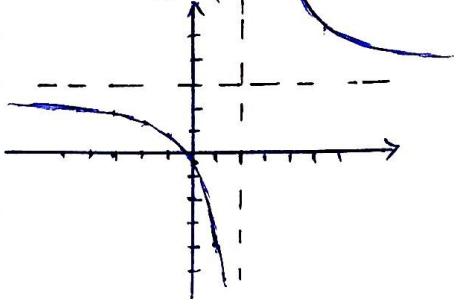
$$f = \{(2, 2), (2, 3), (4, 4)\} \quad h = \{(1, 2), (2, 3), (3, 1), (2, 1)\} \quad g = \{(2, 1), (2, 3), (4, 2)\}$$

$$g^{-1} = \{(1, 2), (3, 2), (2, 1)\} \rightarrow f \circ g^{-1} = \{(1, 2), (2, 3), (2, 1)\}$$

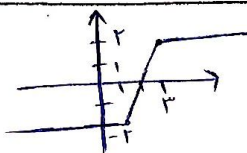
$$\rightarrow \frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{1}{2}\right), \left(2, \frac{1}{2}\right) \right\}$$

$$f(x) = \frac{x^2+1}{x-2}$$

$$x = \frac{y+1}{y-2} \rightarrow f^{-1} = \frac{y+1}{y-2}$$



$$f(x) = |x-1| - |x-2|$$



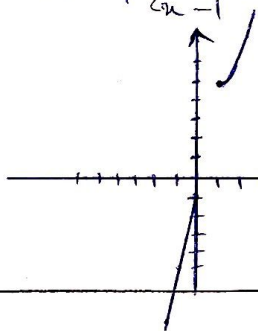
در بازه $[1, 2]$ دارون پیوسته است

$$x \in [1, 2] \rightarrow y = |x-1| - |x-2| = x-1 - (2-x) = 2x-2 \rightarrow x = \frac{y+2}{2} \rightarrow f^{-1} = \frac{y+2}{2}$$

$$f(x) = \begin{cases} x^2+2 & x \geq 1 \\ 2x-1 & x < 1 \end{cases}$$

$$y = x^2+2 \rightarrow x^2 = y-2 \rightarrow x = \sqrt{y-2}$$

$$y = 2x-1 \rightarrow 2x = y+1 \rightarrow x = \frac{y+1}{2}$$



$$f^{-1} = \begin{cases} \sqrt{y-2} & y \geq 3 \\ \frac{y+1}{2} & y < 3 \end{cases}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2(x+2) - (x^2+2x+1)}{x+2} = \frac{-x-1}{x+2}$$

$$\rightarrow x = \frac{-y-1}{y+2} \rightarrow f^{-1} = \frac{-y-1}{y+2} = \frac{-y-1}{y+2} = \frac{ax+b}{cx+d} \rightarrow \begin{cases} a=-1 \\ b=-1 \\ c=1 \\ d=2 \end{cases}$$

$$\rightarrow f^{-1}(\frac{1}{2}) = \frac{1}{2} = 0$$

$$f(x) = \frac{x}{x^2+1}$$

این تابع یک به یک نیست به تابع دارون ندارد

$$f(1) = \frac{1}{2}$$

به مثال دیگر

$$f(\frac{1}{x}) = \frac{1}{\frac{1}{x}+1} = \frac{x}{1+x} = \frac{1}{\frac{1}{x}+1}$$

الته ممکنه ضابطه آن را بدوای به صورت $yx^2 - x + y = 0$ نوشت که یک تابع نباشد $y = \frac{1 \pm \sqrt{1-4xy}}{2x}$