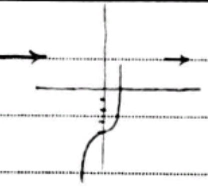
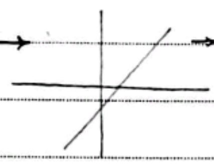
۱- $\begin{cases} x^3-4 & x>a \\ 12x-20 & x<a \end{cases}$



→ یک به یک



→ یک به یک

$x^3-4 \geq 12x-20$

$x^3-12x+16 \geq 0 \Rightarrow x^3-4x-8x+16 \geq 0$

$x(x^2-4)-8(x-2) \geq 0 \rightarrow (x-2)^2(x+4) \geq 0$

	-4		2	
-	0	+	0	+

$\rightarrow [-4, +\infty)$

۲- $f(x)=3x+k \quad f^{-1}(2)=4 \rightarrow f(4)=2 , f(x)=3x+k \rightarrow 12+k=2 \Rightarrow k=-10$

$f(7)=3(7)-10=11 , f(f(x)) \rightarrow f(3x-10) , f(f(x))=3(3x-10)-10 \rightarrow 9x-30-10=9x-40$

۳- $f(x)=\frac{ax}{x-1}, A=(2a,a) \rightarrow f(a)=\frac{a^2}{a-1}=2a \rightarrow a^2=2a^2-2a \rightarrow a^2-2a=0 \rightarrow a(a-2)=0$

$a=2$, $a=0$ غ ق ق

یک به یک x

۴- $fof^{-1} = 5\rightarrow3\rightarrow5 , 7\rightarrow4\rightarrow7 , 2\rightarrow7\rightarrow2 , 6\rightarrow9\rightarrow6 \Rightarrow \{(5,5),(7,7),(2,2),(6,6)\}$

$f^{-1}of = 3\rightarrow5\rightarrow3 , 4\rightarrow7\rightarrow4 , 7\rightarrow2\rightarrow7 , 9\rightarrow6\rightarrow9 \Rightarrow \{(3,3),(4,4),(7,7),(9,9)\}$

$fog^{-1} = 2\rightarrow3\rightarrow5 , 6\rightarrow8\rightarrow x , 5\rightarrow9\rightarrow6 \Rightarrow \{(2,5),(5,6)\}$

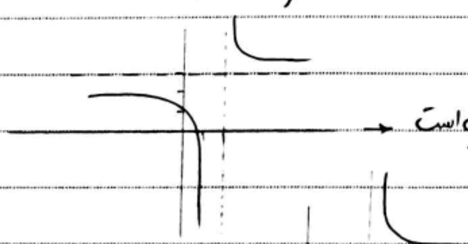
$g^{-1}of = 3\rightarrow5\rightarrow9 , 4\rightarrow7\rightarrow x , 7\rightarrow2\rightarrow3 , 9\rightarrow6\rightarrow8 \Rightarrow \{(3,9),(7,3),(9,8)\}$

۵-

$fog^{-1} = 1\rightarrow2\rightarrow4 , 3\rightarrow4\rightarrow8 , 5\rightarrow6\rightarrow0 \Rightarrow \{(1,4),(3,8),(5,0)\}$

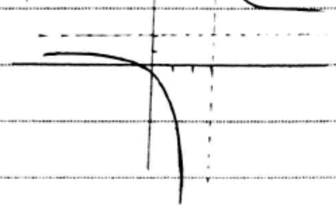
$\frac{h}{fog^{-1}} = \{(1,\frac{2}{4}),(3,\frac{4}{8}),(5,\frac{1}{0})\}$ (third pair crossed out)

۶- $y=\frac{3x+1}{x-2} \rightarrow 3,2$



→ یک به یک است

$f^{-1}(x)=\frac{2x+1}{x-3} \rightarrow 2,3$



$f(x) = |x-1| - |x-3|$ ۷-

	1		3	
$-x+1+x-3 = -2$		$x-1+x-3 = 2x-4$		$x-1-x+3 = 2$
✗	-2	✓	2	✗

$\to [1,3] \to [a,b]$

$x=1 \to f(x) = -2$

$x=3 \to f(x) = 2$

$a=1,\ b=3$

$f^{-1}(x) = \frac{x+4}{2} = \frac{1}{2}x + 2 \to [-2,2]$

$f(x) \begin{cases} x^3+4 \to x \geq 1 \\ 4x-1 \to x \leq 0 \end{cases} \to$ اشتراک بردهای دو ضابطه $\to [5,+\infty)$, $\to (-\infty,-1]$ $\to \varnothing = \cap$ ۸-

$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4}, & x \geq 5 \\ \frac{1}{4}x + \frac{1}{4} & x \leq -1 \end{cases}$

$f(x) = x^2 - \frac{x^3+3x^2+3x+1}{x+3} \to f(x) = \frac{-3x-1}{x+3}$ ۹-

$f^{-1}(x) = \frac{-3x-1}{x+3} = \frac{-6x-2}{2x+6} = \frac{ax+b}{2x+d} \to a=-6,\ b=-2,\ d=6$

$f(x) = \frac{x}{x^2+1}$ → $\frac{x_1}{x_1^2+1}$, $\frac{x_2}{x_2^2+1}$ $\Big\}$ $\frac{x_2}{x_2^2+1} = \frac{x_1}{x_1^2+1} \Rightarrow x_1x_2^2 + x_1 = x_1^2x_2 + x_2$ ۱۰-

$x_1x_2^2 - x_1^2x_2 + x_1 - x_2 = 0$

$x_1x_2(x_2-x_1) - (x_2-x_1) = 0$

$\left.\begin{matrix} x_1 = x_2 \\ x_1x_2 = 1 \end{matrix}\right\}$ یک‌به‌یک نیست