

نام و نام خانوادگی علم و ادب پاسخنامه تشریحی تکلیف شماره ... 11 ... کلاس ... بازرهم ...

برای این تابع یک به یک باشد باید $f(n)$ همواره مقبوضه باشد.

$$f(n) = \begin{cases} n^2 - 4 & ; n > a \\ 12n - 40 & ; n \leq a \end{cases}$$

$$\begin{aligned} a^2 - 4 &\geq 12a - 40 \\ a^2 - 12a + 14 &\geq 0 \\ (a-2)^2(a+4) &\geq 0 \end{aligned}$$

$$\frac{-2}{-2} \pm \frac{2}{2}$$

$$\begin{aligned} a &\geq -1 \\ a &= [-4, +\infty) \end{aligned}$$

2

$$\begin{aligned} y = 3x + k &\Rightarrow 3y + k = x \Rightarrow 3y = x - k \Rightarrow y = \frac{x-k}{3} \Rightarrow f^{-1}(x) = \frac{x-k}{3} \\ f^{-1}(2) = \frac{2-k}{3} = 4 &\Rightarrow 2-k = 12 \Rightarrow k = -10 \Rightarrow f(x) = 3x - 10 \end{aligned}$$

2

$$f(7) = 3 \times 7 - 10 = 11$$

$$\begin{aligned} f(f(x)) &= 3(3x-10) - 10 = 9x - 30 - 10 = 9x - 40 \end{aligned}$$

$$\begin{aligned} y = \frac{am}{n-1} &\Rightarrow n = \frac{ay}{y-1} \Rightarrow my - n = ay \Rightarrow ay - ny = -n \Rightarrow y(a-n) = -n \\ y = \frac{-n}{a-n} &\Rightarrow f^{-1}(n) = \frac{-n}{a-n} \end{aligned}$$

2

$$f^{-1}(4a) = \frac{-4a}{a-4a} = a \Rightarrow \frac{-4a}{-3a} = \frac{4}{3} \Rightarrow a = \frac{4}{3}$$

$$\begin{aligned} f \circ f^{-1} &= \{(2,3)(4,2)(6,4)(8,6)\} \\ f \circ f^{-1} &= \{(2,2)(4,4)(6,6)(8,8)\} \end{aligned}$$

$$\begin{aligned} f^{-1} \circ f &= \{(3,2)(2,4)(4,6)(6,8)\} \\ f^{-1} \circ f &= \{(3,3)(2,2)(4,4)(6,6)\} \end{aligned}$$

1, 1, 1, 1, 1, 1, 1, 1

$$\begin{aligned} f \circ g^{-1} &= \{(2,5)(5,7)\} \\ f \circ g^{-1} &= \{(2,2)(5,5)\} \end{aligned}$$

$$\begin{aligned} g^{-1} \circ f &= \{(3,1)(1,9)(9,3)\} \\ g^{-1} \circ f &= \{(3,3)(1,1)(9,9)\} \end{aligned}$$

$$g^{-1} = \{(1,2)(3,4)(5,6)\} / f \circ g^{-1} = \{(1,2)(3,4)(5,6)\}$$

$$h \circ g^{-1} = \{(1,1)(3,1)(5,1)\}$$

2

5

$y = \frac{2n+1}{n-2} \rightarrow y(n-2) = 2n+1$ $yn - 2y = 2n+1$ $yn - 2y = 2n+1$ $n = \frac{2y+1}{y-2}$ $y = \frac{2n+1}{n-2} \rightarrow f^{-1}(n) = \frac{2n+1}{n-2}$ ✓	$f^{-1}(n)$		۶
$f(n) = n-1 - n-2 $ 	$[a, b] \Rightarrow [1, 2]$ $f(n) = 2n-4 \rightarrow y = 2n-4 \Rightarrow n = \frac{y+4}{2}$ $2y = n+4 \rightarrow y = \frac{n+4}{2} \rightarrow f^{-1}(n) = \frac{n+4}{2}$ $D_{f^{-1}}: [-4, 2]$ ✓ $R_{f^{-1}}: [1, 2]$ ✓		۷
$f(n) = \begin{cases} n^2+4 & ; n \geq 1 \\ n-1 & ; n \leq 0 \end{cases}$ 	$n^2+4 = y$ $y = n-1 \rightarrow n = y+1$ $f^{-1}(n) = \begin{cases} \sqrt{n-4} & ; n \geq 4 \\ \frac{n+1}{2} & ; n < 4 \end{cases}$ $R_{f(n)} = [4, \infty) \cup (-1, \infty)$ $R_{f^{-1}(n)} = [4, \infty) \cup (-1, \infty)$ ✓ $f(n-1) = y$ $fy = n+1$ $y = \frac{n+1}{f}$		۸
$f(n) = n^2 - \frac{(n+1)^2}{(n+2)}$ $\Rightarrow \frac{n^2(n+2) - (n+1)^2}{n+2} = \frac{n^3+2n^2 - (n^2+2n+1)}{n+2} = \frac{n^3+n^2-2n-1}{n+2}$ $\frac{-2n-1}{n+2} \Rightarrow f^{-1}(n) = \frac{-2n-1}{n+2} = \frac{-2n-2}{n+2} \rightarrow a = -2, b = -2, d = 2$ $f^{-1}(-1) = \frac{(-4(-1)-2)}{(-1+2)} = \frac{4-2}{1} = 2$ ✓	$f^{-1}(n) = \frac{-2n-1}{n+2}$ $f^{-1}(-1) = \frac{(-4(-1)-2)}{(-1+2)} = \frac{4-2}{1} = 2$ ✓		۹
$f(n) = \frac{n}{n^2+1}$ $y = \frac{n}{n^2+1} \rightarrow n = \frac{y}{y^2+1} \rightarrow ny^2 + n - y = 0$ $y = \frac{+1 \pm \sqrt{1-4(n^2)}}{2n}$ $R_{f(n)} = [-\frac{1}{2}, \frac{1}{2}] \rightarrow D_{f^{-1}(n)} = [-\frac{1}{2}, \frac{1}{2}]$ $f^{-1}(n) = \frac{+1 \pm \sqrt{1-4(n^2)}}{2n}$ ✓ $D_{f^{-1}(n)} = [-\frac{1}{2}, \frac{1}{2}]$ ✓	$R_{f(n)} = [-\frac{1}{2}, \frac{1}{2}] \rightarrow D_{f^{-1}(n)} = [-\frac{1}{2}, \frac{1}{2}]$ $f^{-1}(n) = \frac{+1 \pm \sqrt{1-4(n^2)}}{2n}$ ✓ $D_{f^{-1}(n)} = [-\frac{1}{2}, \frac{1}{2}]$ ✓		۱۰

فقط حالت ۱۰ قابل قبول است زیرا α و β هم علامت اند.