

$y = (a-1)x^2 + x + 3 \rightarrow -1 = f(a-1) = fa - f = -1 \rightarrow fa = 3 \rightarrow a = \frac{3}{f}$
 $\rightarrow -b = 2 \rightarrow -b = fa$
 $\Rightarrow y = (\frac{3}{f}-1)x^2 + x + 3 = -\frac{1}{f}x^2 + x + 3 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+3}}{-\frac{1}{f}} = \frac{-1 \pm 2}{-\frac{1}{f}}$
 $\Rightarrow x = \frac{-3}{-\frac{1}{f}} \rightarrow x = 4$ ✓
 $\Rightarrow x = \frac{1}{-\frac{1}{f}} \rightarrow x = -f \rightarrow x \text{ غرضی}$

$f(x) = mx^2 - 5x + m$
 $\Delta > 0 \rightarrow 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \rightarrow m^2 < \frac{25}{4}$
 $\Rightarrow -\frac{5}{2} < m < \frac{5}{2}$
 $m > 0$
 $\Rightarrow m \in (-\frac{5}{2}, 0)$

$\alpha = \frac{3-\sqrt{5}}{2}$
 $\beta = \frac{3+\sqrt{5}}{2}$
 $\alpha + \beta = 3 = -\frac{b}{a} \rightarrow \frac{3-\sqrt{5}}{2} + \frac{3+\sqrt{5}}{2} = \frac{6}{2} = 3$
 $\alpha \beta = \frac{c}{a} \rightarrow (\frac{3-\sqrt{5}}{2})(\frac{3+\sqrt{5}}{2}) = \frac{9-5}{4} = \frac{4}{4} = 1$
 $\Rightarrow x^2 - 3x + 1 = 0 \rightarrow x^2 - 3x + \frac{4}{4} = 0 \rightarrow 4x^2 - 12x + 4 = 0$

$2x^2 - 11x + m + 2 = 0 \rightarrow 2x^2 - 11x + m + 2 = 0 \rightarrow 2x^2 = 11x - m - 2$
 $\alpha < \beta$
 $2\alpha^2 + \beta^2 = 11$
 $\Rightarrow \alpha^2 + \beta^2 + \alpha^2 = 5 - 2p + 6x^2 \rightarrow 5 - 2p + 11\alpha - m - 2 = 11$
 $\Rightarrow 3 - 2p = 14 - m - 2 = 12 - m$
 $\Rightarrow 12 - m + 11\alpha - m - 2 = 11 \rightarrow 11\alpha = 2m \rightarrow \alpha = \frac{2m}{11}$
 $\Rightarrow \alpha = \frac{2m}{11} \rightarrow \beta = \frac{11-m}{2}$
 $\Rightarrow \alpha + \beta = 3 \rightarrow \frac{2m}{11} + \frac{11-m}{2} = 3 \rightarrow 4m + 121 - 11m = 66 \rightarrow -7m = -55 \rightarrow m = 7$

$S = (2, 9)$
 $y = a(x-h)^2 + k \rightarrow y = a(x-2)^2 + 9$
 $\Rightarrow y = a(x^2 - 4x + 4) + 9 = ax^2 - 4ax + 4a + 9 = 0$
 $\Rightarrow C = 5 = 4a + 9 \rightarrow a = -1$
 $\Rightarrow y = -x^2 + 4x + 13 = 0 \rightarrow x^2 - 4x - 13 = 0$
 $\Rightarrow x = 1$
 $\Rightarrow x = 5$
 $\Rightarrow -1 < m < 5$
 $m \in (-1, 5)$

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$$\alpha x^2 - \gamma x + 9\beta = 0 \rightarrow \begin{cases} S = \alpha + \beta = \frac{\gamma}{\alpha} \\ P = \frac{9\beta}{\alpha} \end{cases}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = S \rightarrow \frac{\alpha + \beta}{\alpha\beta} = S \rightarrow \frac{\frac{\gamma}{\alpha}}{\frac{9\beta}{\alpha}} = \frac{\gamma}{9\beta} = \frac{\gamma}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} \rightarrow \gamma = 9\alpha + 9\beta$$

$$\Rightarrow -\gamma\alpha = 9\beta \leftarrow \begin{cases} \alpha\beta^2 - \gamma\beta + 9\alpha = \alpha\beta^2 + \gamma\beta = \beta(\alpha\beta + \gamma) = 0 \rightarrow \begin{cases} \beta = 0 \\ \alpha\beta = P = -\gamma \end{cases} \end{cases}$$

$$\Rightarrow P = \frac{9\beta}{\alpha} = -\gamma \Rightarrow \alpha = -\frac{\gamma}{P}\beta \rightarrow S = \alpha + \beta = -\frac{\gamma}{P}\beta \Rightarrow P = \alpha\beta = (-\frac{\gamma}{P}\beta)(\beta) = -\frac{\gamma}{P}\beta^2 = P = -\gamma \Rightarrow \beta^2 = -P \Rightarrow \beta = \frac{\gamma}{P} \rightarrow \frac{S}{P} = \frac{-\gamma}{\frac{\gamma}{P}} = -P$$

$$\alpha^2 + m\alpha - \gamma m = 0 \xrightarrow{\alpha} \alpha^2 + m\alpha - \gamma m = 0 \rightarrow \alpha^2 = -m\alpha + \gamma m$$

$$\alpha^2 - m\beta = \lambda \xrightarrow{\alpha^2 = -m\alpha + \gamma m} -m\alpha + \gamma m - m\beta = \lambda \rightarrow -m(\alpha + \beta) + \gamma m - \lambda = 0$$

$$S = \alpha + \beta = ? \Rightarrow \begin{cases} S = -m \\ P = -\gamma m \end{cases} \Rightarrow m^2 + \gamma m - \lambda = 0 \rightarrow (m + \gamma)(m - \gamma) = 0 \rightarrow m = -\gamma \rightarrow \Delta > 0 \checkmark$$

$$f(m) = m\alpha^2 + \gamma m + \frac{m^2}{\gamma} + \gamma \xrightarrow{\frac{m^2}{\gamma} + \gamma m} \text{ext} = -\frac{\Delta}{f_a} = \frac{-(14 - f(m)(\frac{m}{\gamma} + \gamma))}{f_m} = \frac{-14 + \gamma m^2 + \gamma \lambda m}{f_m} = \frac{m^2 + \gamma m - \lambda}{\gamma m} = \lambda$$

$$\Rightarrow m^2 + \gamma m - \lambda = 14m \rightarrow m^2 - \gamma m - \lambda = 0 \rightarrow (m - \gamma)(m + \gamma) = 0$$

$$\begin{cases} m = \gamma \rightarrow f_m^2 + f_m + 9 \rightarrow X \\ m = -\gamma \rightarrow -\gamma\alpha^2 + \gamma\alpha + 4 \xrightarrow{m = -\gamma} (f_m) = \alpha = \frac{-f \pm \sqrt{14 + f\lambda}}{-f} \end{cases}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = S \xrightarrow{\text{مضرب}} \frac{\alpha + \beta}{\alpha\beta} = \frac{S}{P} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c} = S \rightarrow \frac{-\gamma}{F} = -\frac{1}{F}$$

$$\alpha x^2 + bx + c \rightarrow \begin{cases} C = F \rightarrow \text{مضرب} \\ -\frac{b}{\gamma a} = \gamma \rightarrow -b = \gamma a \\ -\frac{\Delta}{f_a} = \frac{b^2 - fac}{-fa} = 4 \rightarrow b^2 - fac = -\gamma fa \xrightarrow{-b = \gamma a} b^2 + bc = 4b \end{cases}$$

$$\Rightarrow \begin{cases} b + c = 4 \\ c = F \end{cases} \rightarrow b = \gamma \Rightarrow \frac{-b}{c} = \frac{-\gamma}{F} = -\frac{1}{F}$$

$$\alpha^2 - (fa + f)x + (3a^2 + 4a + 3) = 0$$

$$\alpha = 1 \rightarrow x^2 - 11x + 12 \rightarrow \checkmark$$

$$\alpha = -\frac{1}{\gamma} \rightarrow x^2 - \frac{14}{\gamma}x + \gamma \rightarrow x = \frac{\frac{14}{\gamma} \pm \sqrt{\frac{196}{\gamma^2} - 4}}{2} = \frac{14 \pm \sqrt{196 - 4\gamma^2}}{2\gamma}$$

$$\Rightarrow x^2 - 11x + 12 \rightarrow x = \frac{11 \pm \sqrt{121 - 48}}{2} = \frac{11 \pm 5}{2} \rightarrow x = 8 \text{ or } x = 3$$