علیرضا ... یازدهم ۸ پسر سری ۱۳

۲۰ آفرین!

۱) **(۲)**

$y=(a-1)x^2+x+3$

$x=2$ تقارن $\rightarrow \frac{-b}{2a}=2 \rightarrow b=-4a$

$\rightarrow 1=4-4a$

$\rightarrow a=\frac{3}{4} \rightarrow$ ضریب x^2 $=\frac{-1}{4} \rightarrow -\frac{1}{4}x^2+x+3=0$

$x^2-4x-12=0$

$x \begin{cases} 6 \checkmark \\ -2 \text{ غق} \end{cases}$ ✓

ریشه‌ی مثبت؟ $x=6$

۲) **(۲)**

$f(x)=mx^2-5x+m$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

ضریب $x^2<0$

$m<0$

۲ ریشه $\rightarrow \Delta>0 \rightarrow 25-4m^2>0$

$m^2<\frac{25}{4} \rightarrow -\frac{5}{2}<m<\frac{5}{2}$

$m<0$

$\rightarrow -\frac{5}{2}<m<0$ ✓

$m=?\ (-\frac{5}{2}, 0)$

۳) **(۲)**

معادله درجه دوم - ضرایب صحیح

$ax^2+bx+c=0$

$\alpha, \beta = \frac{3\pm\sqrt{5}}{3}$

$S=\frac{-b}{a}=\alpha+\beta=\frac{3+\sqrt{5}+3-\sqrt{5}}{3}=2 \rightarrow b=-2a$

$P=\frac{c}{a}=\alpha\times\beta=\frac{(3+\sqrt{5})(3-\sqrt{5})}{3\times3}=\frac{4}{9} \rightarrow c=\frac{4a}{9}$

$ax^2-2ax+\frac{4}{9}a$

ضرایب صحیح

✓ $9x^2-18x+4=0$

۴) **(۲)**

$2x^2-8x+m+2=0$

α, β ریشه

$\alpha<\beta$

$3\alpha^2+\beta^2=12 \rightarrow 2\alpha^2+\alpha^2+\beta^2=12 \xrightarrow[\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta]{2\alpha^2=8\alpha-m-2}$ $(\alpha+\beta)^2-2\alpha\beta+8\alpha-m-2=12$

(α در معادله صدق می‌کند؛ $\frac{8}{2}=4$، $\frac{m+2}{2}$)

$14-2m+8\alpha=14 \rightarrow m=4\alpha$

α در معادله صدق می‌کند

$2\alpha^2-8\alpha+4\alpha+2=0$

$\alpha^2-2\alpha+1=0 \rightarrow \alpha=1$

$m=?=4\alpha=4$ ✓

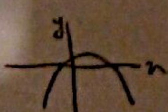
۵) **(۲)**

$Ext\,|^2_9$ $\quad y=ax^2+bx+c$

$c=5$ $\quad \frac{-b}{2a}=2 \rightarrow b=-4a$

$4a+2b+c=9,\ c=5 \rightarrow 2a+b=2,\ b=-4a \rightarrow -2a=2,\ a=-1 \rightarrow b=4 \rightarrow y=-x^2+4x+5$

ریشه $x=5, -1$



{x| بالای محور} $=?$ $\rightarrow$ تابع ماکسیمم دارد $\rightarrow$ بین دو ریشه از x بالاتر $\rightarrow$ پاسخ: $(-1, 5)$ ✓

Scanned by CamScanner

$x^2 - 7x + 9\beta = 0$

$\alpha, \beta \in \{2, \beta\}$.

$P = \frac{c}{a} = \alpha\beta = \frac{9\beta}{\alpha} \rightarrow \beta = 0$ ✗

$\alpha = \frac{9}{\alpha} \rightarrow \alpha \in \{3, -3\}$

$S = \frac{-b}{a} = \alpha + \beta = \frac{7}{\alpha} \rightarrow \begin{cases} \alpha = 3 & \beta = \frac{-3}{3} \times \\ \alpha = -3 & \beta = \frac{3}{3} \checkmark \end{cases}$

$\frac{1}{\alpha} + \frac{1}{\beta} = ?$ $\quad \frac{1}{-3} + \frac{1}{\frac{3}{3}} = \frac{+7}{9}$ دقت!

(۲)

$x^2 + mx - 3m = 0 \rightarrow \Delta > 0 \rightarrow m^2 + 8m > 0 \xrightarrow{\frac{-8}{+1} = -8} m \in (-\infty, -8) \cup (0, +\infty)$

$m = \begin{cases} 3 & \Delta > 0 \checkmark \\ -6 & \Delta < 0 \times \end{cases}$

$\alpha, \beta \in \{2, \ldots\} \xrightarrow{\text{جایگذاری}} \alpha^2 + \alpha m - 3m = 0$

$\alpha^2 = 3m - \alpha m$

$\alpha^2 - m\beta = 8 \rightarrow 3m - m\alpha - m\beta = 8$

$m(3 - (\alpha + \beta)) = 8 \xrightarrow{\alpha + \beta = \frac{-b}{a} = \frac{-m}{1}} m\left(3 - \left(\frac{-m}{1}\right)\right) = 8 \rightarrow m^2 + 3m - 18 = 0 \rightarrow m = \begin{cases} 3 \\ -6 \end{cases}$

$\alpha + \beta = ?$ $\quad \alpha + \beta = S = \frac{-b}{a} = \frac{-m}{1} = -3$ ✓

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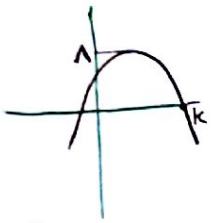
$f(x) = mx^2 + 6x + \frac{m}{3} + 7$

سهمی ماکسیمم دارد $\rightarrow a < 0 \rightarrow m < 0$



$\frac{-\Delta}{4a} = 8 \rightarrow \Delta = -32a$

$b^2 - 4ac = -32a$

$144 - 12m^2 - 24m = -32m \rightarrow m^2 - 2m + 8 = 0$ $\quad m \in \begin{cases} 6 & a > 0 \times \\ -4 & \checkmark \end{cases}$

$\rightarrow f(x) = -4x^2 + 6x + 7 = 0$

$\rightarrow x \in \{-1, \frac{7}{4}\}$

$k = ?$ $\quad$ مساحت $= 9$ ✓

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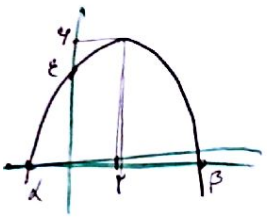

$y = ax^2 + bx + 6$

$\frac{-b}{2a} = 2 \rightarrow b = -4a$

$\begin{cases} y = ax^2 - 4ax + 6 \\ \max \begin{cases} 2 \\ 4 \end{cases} \end{cases} \rightarrow 4 = 4a - 8a + 6$

$a = \frac{-1}{2} \rightarrow y = \frac{-1}{2}x^2 + 2x + 6$

$\rightarrow$ معادله: $x^2 - 4x - 8 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} = ?$ $\quad \frac{\alpha + \beta}{\alpha\beta} = \frac{S}{P} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c} = \frac{4}{-8} = \frac{-1}{2}$ ✓

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$x^2 - (4a + 4)x + (2a^2 + 4a + 5) = 0$

$\alpha, \beta \in \Omega, \rightarrow \Delta > 0 \rightarrow 16a^2 + 32a + 16 - 12a^2 - 24a - 12 > 0 \rightarrow a^2 + 2a + 1 > 0 \rightarrow a \neq -1$

$\alpha = 2$ جایگذاری $\quad 4 - 2(4a + 4) + 2a^2 + 4a + 5 = 0 \rightarrow 2a^2 - 2a - 1 = 0 \rightarrow a \in \begin{cases} 1 \\ \frac{-1}{2} \end{cases}$

$a = 1 \rightarrow x^2 - 8x + 12 = 0 \rightarrow x \in \begin{cases} 2 \\ 6 \end{cases}$ ✓

$a = \frac{-1}{2} \rightarrow x^2 - \frac{8}{2}x + \frac{4}{2} = 0 \rightarrow x \in \begin{cases} 2 \\ \frac{7}{2} \end{cases}$ ✓

$\beta = ?$ $\quad \beta \in \begin{cases} 6 \\ \frac{7}{2} \end{cases}$

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