۱)

$y=(a-1)x^2+x+3$

$x=2$ تقارن $\rightarrow \frac{-b}{2a}=2 \rightarrow b=-4a$

$\rightarrow 1=4-4a$

$\rightarrow a=\frac{3}{4} \rightarrow$ ضریب $x^2=\frac{-1}{4} \rightarrow -\frac{1}{4}x^2+x+3=0$

$x^2-4x-12=0$, $x \begin{cases} 6 \checkmark \\ -2 \end{cases}$ غ ق ق

ریشه مثبت؟ $x=6$

۲)

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$f(x)=mx^2-5x+m$

ضریب $x^2<0$

$m<0$

۲ ریشه $\rightarrow \Delta>0 \rightarrow 25-4m^2>0$

$m^2<\frac{25}{4} \rightarrow -\frac{5}{2}<m<\frac{5}{2}$

$m<0$

$\rightarrow -\frac{5}{2}<m<0$

$m=?\quad (-\frac{5}{2}, 0)$

۳)

معادله درجه دوم - ضرایب صحیح

$ax^2+bx+c=0$

$\boxed{9x^2-18x+4=0}$

ضرایب صحیح

$\alpha, \beta=\frac{3\pm\sqrt{5}}{3}$

$S=\frac{-b}{a}=\alpha+\beta=\frac{3+\sqrt{5}+3-\sqrt{5}}{3}=2 \rightarrow b=-2a$

$P=\frac{c}{a}=\alpha\times\beta=\frac{(3+\sqrt{5})(3-\sqrt{5})}{3\times 3}=\frac{4}{9} \rightarrow c=\frac{4a}{9}$

$\rightarrow ax^2-2ax+\frac{4}{9}a$

۴)

$2x^2-8x+m+2=0$

α, β ریشه

$\alpha<\beta$

α در معادله صدق می کند

$3\alpha^2+\beta^2=12 \rightarrow 2\alpha^2+\alpha^2+\beta^2=12 \xrightarrow[\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta]{2\alpha^2=8\alpha-m-2} (\overbrace{\alpha+\beta}^{\frac{8}{2}=4})^2-2\overbrace{\alpha\beta}^{\frac{m+2}{2}}+8\alpha-m-2=12$

$16-2m+8\alpha=16 \rightarrow m=4\alpha$

$2\alpha^2-8\alpha+4\alpha+2=0$

$\alpha^2-2\alpha+1=0 \rightarrow \alpha=1$

α در معادله صدق می کند

$m=?=4\alpha=4$

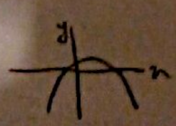
۵)

$Ext\Big|^{2}_{9}$ $\quad y=ax^2+bx+c$

$c=5$ $\quad \frac{-b}{2a}=2 \rightarrow b=-4a$

$\begin{cases} 4a+2b+c=9 \\ c=5 \end{cases} \quad \begin{cases} b=-4a \\ 2a+b=2 \end{cases} \quad -2a=2,\ a=-1 \rightarrow b=4 \rightarrow y=-x^2+4x+5$

ریشه ها $x=5, -1$



x بالای محور = ?

تابع ماکسیمم دارد $\longrightarrow$ بین دو ریشه از محور x ها بالاتر $\longrightarrow$ پاسخ: $(-1, 5)$

۶

$\alpha x^2 - 7x + 9\beta = 0$

α, β ریشه، $\beta > 0$

$P = \frac{c}{a} = \alpha \times \beta = \frac{9\beta}{\alpha} \rightarrow \begin{cases} \beta = 0 \; ✗ \\ \alpha = \frac{9}{\alpha} \rightarrow \alpha \begin{cases} 3 \\ -3 \end{cases} \end{cases}$

$S = \frac{-b}{a} = \alpha + \beta = \frac{7}{\alpha} \rightarrow \begin{cases} \alpha = 3 & \beta = \frac{-2}{3} \; ✗ \\ \alpha = -3 & \beta = \frac{2}{3} \; ✓ \end{cases}$

$\frac{1}{\alpha} + \frac{1}{\beta} = ? \quad \frac{1}{-3} + \frac{1}{\frac{2}{3}} = \frac{-7}{6}$

۷

$x^2 + mx - 2m = 0 \rightarrow \Delta > 0 \rightarrow m^2 + 8m > 0 \quad \frac{-8 \quad 0}{+ \quad - \quad +} \rightarrow m : (-\infty, -8) \cup (0, +\infty)$

α, β ریشه $\xrightarrow{\text{ریشه در معادله صدق میکند } \alpha}$ $\alpha^2 + \alpha m - 2m = 0$

$\alpha^2 = 2m - \alpha m$

$\alpha^2 - m\beta = 8 \rightarrow 2m - m\alpha - m\beta = 8$

$m(2 - (\alpha+\beta)) = 8 \xrightarrow{\alpha+\beta = \frac{-b}{a} = -m} m\left(2 - \left(-m\right)\right) = 8 \rightarrow m^2 + 2m - 8 = 0 \rightarrow m = \begin{cases} 2 \\ -4 \end{cases}$

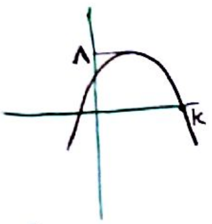
$m = \begin{cases} 2 & \Delta > 0 \; ✓ \\ -4 & \Delta < 0 \; ✗ \end{cases}$

$\alpha + \beta = ? \quad \alpha + \beta = S = \frac{-b}{a} = \frac{-m}{1} = -2$

۸

$f(x) = mx^2 + 4x + \frac{m}{2} + 7$

سهمی max دارد $\rightarrow a < 0 \rightarrow m < 0$



$\frac{-\Delta}{4a} = 8 \rightarrow \Delta = -32a$

$b^2 - 4ac = -32a$

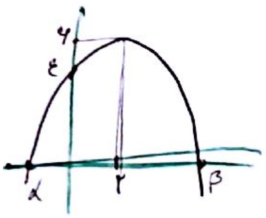
$16 - 2m^2 - 28m = -32m \rightarrow m^2 - 2m - 8 = 0 \quad m \begin{cases} 4 & a > 0 \; ✗ \\ -2 & ✓ \end{cases}$

$\rightarrow f(x) = -2x^2 + 4x + 6 = 0$

$\rightarrow x = -1, 3$

$k = ? =$ ریشه مثبت $= 3$

۹



$y = ax^2 + bx + 4$

$\left.\begin{aligned} &\frac{-b}{2a} = 2 \rightarrow b = -4a \end{aligned}\right\} y = ax^2 - 4ax + 4 \quad \max \begin{vmatrix} 2 \\ 6 \end{vmatrix} \Big\} \; 6 = 4a - 8a + 4$

$a = \frac{-1}{2} \rightarrow y = \frac{-1}{2}x^2 + 2x + 4$

$\rightarrow$ معادله: $x^2 - 4x - 8 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} = ? \quad \frac{\alpha+\beta}{\alpha\beta} = \frac{S}{P} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c} = \frac{4}{-8} = \frac{-1}{2}$

۱۰

$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0$

α, β ریشه $\rightarrow \Delta > 0 \rightarrow 16a^2 + 32a + 16 - 12a^2 - 24a - 12 > 0 \rightarrow a^2 + 2a + 1 > 0 \rightarrow a \neq -1$

$\alpha = 2 \xrightarrow{\text{در معادله صدق میکند}} 4 - 2(4a+4) + 3a^2 + 6a + 3 = 0 \rightarrow 3a^2 - 2a - 1 = 0 \rightarrow a \begin{cases} 1 \\ \frac{-1}{3} \end{cases}$

$a = 1 \rightarrow x^2 - 8x + 12 = 0 \rightarrow x \begin{cases} 2 \\ 6 \end{cases}$

$\beta = ? \quad \beta \begin{cases} 6 \\ \frac{2}{3} \end{cases}$

$a = \frac{-1}{3} \rightarrow x^2 - \frac{8}{3}x + \frac{4}{3} = 0 \rightarrow x \begin{cases} 2 \\ \frac{2}{3} \end{cases}$