

$$m_{AB} = \frac{1 - (-4)}{2 - 4} = -1 \Rightarrow m_{\text{عمودي}} = \frac{1}{1}$$

$$AB \text{ خط } \rightarrow \left(\frac{4+2}{2}, \frac{1+(-4)}{2} \right) = (3, 1) \Rightarrow y - 1 = \frac{x - 3}{1} \Rightarrow y = x - 2 \rightarrow x - y - 3 = 0$$

$$BC = \sqrt{(3-4)^2 + (-1-(-4))^2} = \Delta$$

$$BC: m = \frac{3-4}{-1-(-4)} = -\frac{1}{3} \quad y - 3 = -\frac{1}{3}x(x - (-1)) \Rightarrow y = -\frac{1}{3}x + \frac{8}{3}$$

$$\Rightarrow 3y + x - 8 = 0 \quad AH = \frac{|12 + 3 - 8|}{\sqrt{14 + 9}} = 2 \Rightarrow BC - AH = \Delta - 2 = 3$$

قطر دائره = نصف قطر

$$2y - x = 14 \rightarrow 4y - 2x = 14 \quad \begin{cases} 4y - 2x = 14 \\ 4y - 2x = 2 \end{cases} \rightarrow a = 2$$

$$d = \frac{|14 - 2|}{\sqrt{16 + 4}} = \frac{12}{\sqrt{20}} \Rightarrow شعاع = \frac{6}{\sqrt{20}}$$

$$S = \pi \times \left(\frac{6}{\sqrt{20}} \right)^2 = \frac{36}{10} \pi = \frac{9}{5} \pi$$

$$M \left(\frac{2+4}{2}, \frac{3+1}{2} \right) = M(3, 2)$$

$$AB: m = \frac{1-3}{4-2} = -1 \Rightarrow m_{CH} = 1 \quad y - 3 = -(x - 2) \Rightarrow y = -x + 5$$

$$CH: m = 1 \quad y - (-3) = x - (-1) \Rightarrow y = x - 2$$

$$x - 2 = -x + 5 \Rightarrow 2x = 7 \Rightarrow x = \frac{7}{2}, y = \frac{3}{2} \Rightarrow H \left(\frac{7}{2}, \frac{3}{2} \right)$$

$$MH = \sqrt{\left(\frac{7}{2} - \frac{3}{2} \right)^2 + \left(2 - \frac{3}{2} \right)^2} = \sqrt{\frac{1}{2} + \frac{1}{4}} = \sqrt{\frac{3}{4}}$$

$$m_1 = -\frac{3m-2}{2-m}$$

$$m_2 = -\frac{m+1}{-2m+4}$$

$$m_1 m_2 = -1 \Rightarrow \frac{3m-2}{m-2} \times \frac{m+1}{2m-4} = -1$$

$$\Rightarrow 3m^2 + m - 2 = -2m^2 + 14m - 20 \Rightarrow 5m^2 - 13m + 18 = 0 \rightarrow \Delta < 0 \rightarrow \text{لا يوجد حله}$$

$$AB: 2y = -x + 3 \Rightarrow y = \frac{-x+3}{2}$$

$$BC: y = -x + 5$$

$$B \begin{cases} AB \\ BC \end{cases} \rightarrow -x + 5 = \frac{-x+3}{2} \Rightarrow x=0 \quad y=-1 \Rightarrow B(0, -1) \quad (2)$$

$$A \begin{cases} AB \\ AC \end{cases} \rightarrow \frac{-x+3}{2} = 2x-1 \Rightarrow -x+3=4x-2 \Rightarrow x=1 \quad y=1 \quad A(1, 1)$$

$$C \begin{cases} AC \\ BC \end{cases} \rightarrow 2x-1 = -x+5 \Rightarrow x=\frac{6}{3} \quad y=\frac{5}{3} \quad C\left(\frac{6}{3}, \frac{5}{3}\right)$$

$$M\left(\frac{0+0}{2}, \frac{-1+5}{2}\right) = \left(\frac{1}{2}, \frac{2}{2}\right) \Rightarrow AM = \sqrt{\left(1-\frac{1}{2}\right)^2 + \left(1-\frac{2}{2}\right)^2} = \frac{\Delta\sqrt{2}}{2}$$

$$AH = \frac{|1+1-5|}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow \frac{AM}{AH} = \frac{\frac{\Delta\sqrt{2}}{2}}{\sqrt{2}} = \boxed{\frac{\Delta}{2}} \quad \checkmark$$

$$A=(0, y) \quad y = -\frac{1}{2}x + b$$

$$B=(x, 0) \quad \sqrt{2} = \frac{b}{\frac{\sqrt{2}}{2}} \Rightarrow b = \Delta \Rightarrow y = -\frac{x}{2} + \Delta \quad (2)$$

$$AB = \sqrt{1+25} = \boxed{5\sqrt{2}} \quad \checkmark \Rightarrow A(0, \Delta), B(1, 0)$$

$$d_1=0 \Rightarrow -4x = 2m \Rightarrow x = -\frac{m}{2}$$

$$d_2=0 \Rightarrow -mx + x + 2 = 0 \Rightarrow x = \frac{2}{m-1}$$

$$\text{جواب: } 3(-1) = -3 \quad (1, 2)$$

$$\left| \frac{2}{m-1} + \frac{m}{2} \right| = 0 \Rightarrow 4+m^2-m = 0 \Rightarrow m^2-7m+4=0 \Rightarrow m = \frac{7 \pm \sqrt{49-16}}{2} = \frac{7 \pm \sqrt{33}}{2}$$

$$\hookrightarrow m^2 + 4m - 1 = 0 \quad \Delta > 0 \Rightarrow \frac{C}{a} = -1$$

$$\text{خط عمود و گزیده از } A(2, 1)$$

$$\frac{y+1}{2} = 2 \Rightarrow x_{A'} = 2$$

$$\frac{1+y}{2} = -2 \Rightarrow y_{A'} = -5 \Rightarrow A'(2, -5) \quad (2)$$

$$y - (-5) = 2(x - 2) \Rightarrow$$

$$\boxed{y = 2x - 9} \quad \checkmark$$

$$y-2 = -(x-1) \Rightarrow y = -x+3 \rightarrow \text{خط عمود و گزیده از } A(2, 1)$$

$$\begin{cases} y = x+2 \\ y = -x+3 \end{cases} \rightarrow x+2 = -x+3 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}, y = \frac{5}{2}$$

$$\Rightarrow B(-1, 4)$$

$$\frac{1+x}{2} = 1 \Rightarrow x = 1 \Rightarrow A'(-3, 4)$$

$$\frac{2+y}{2} = 4 \Rightarrow y = 6$$

$$\Rightarrow a = -3, b = 4 \Rightarrow 2b - a = 12 - (-3) = \boxed{15} \quad \checkmark$$