

$$m_{AB} = \frac{1 - (-4)}{2 - 4} = -1 \Rightarrow \text{عمود منتهی} = \frac{1}{-1}$$

$$AB \text{ خط مستقیم} \rightarrow \left(\frac{4+2}{2}, \frac{1+(-4)}{2} \right) = (3, 1) \Rightarrow y - 1 = \frac{x - 3}{-1} \Rightarrow y = \frac{x}{-1} + \frac{4}{-1} \rightarrow Vy - x - 4 = 0$$

$$BC = \sqrt{(3-4)^2 + (-1-(-4))^2} = \Delta$$

$$BC: m = \frac{3-4}{-1-(-4)} = \frac{-1}{3} \quad y - 3 = -\frac{1}{3}x(x - (-1))$$

$$\Rightarrow y = -\frac{1}{3}x + \frac{8}{3}$$

$$\Rightarrow 3y + 4x - 8 = 0$$

$$AH = \frac{|12 + 3 - 8|}{\sqrt{16 + 9}} = 2 \Rightarrow BC - AH = \Delta - 2 = \boxed{3}$$

قطر دایره = فاصله دو خط

$$2y - x = 4 \quad 4y - 2x = 14 \quad \begin{cases} 4y - 2x = 14 \\ 4y - 2x = 2 \end{cases} \xrightarrow{\text{تفاضل}} a = 2$$

$$d = \frac{|14 - 2|}{\sqrt{16 + 4}} = \frac{12}{\sqrt{20}} \Rightarrow \text{شعاع} = \frac{6}{\sqrt{20}}$$

$$S = \pi \times \left(\frac{6}{\sqrt{20}} \right)^2 = \frac{36}{20} \pi = \boxed{\frac{9}{5} \pi}$$

$$M \left(\frac{2+4}{2}, \frac{3+1}{2} \right) = M(3, 2)$$

$$AB: m = \frac{1-3}{4-2} = \frac{-2}{2} = -1 \Rightarrow m_{CH} = 1 \quad y - 3 = -(x - 2) \Rightarrow y = -x + 5$$

$$CH: m = 1 \quad y - (-3) = x - (-1) \Rightarrow y = x - 2$$

$$x - 2 = -x + 5 \Rightarrow 2x = 7 \Rightarrow x = \frac{7}{2}, y = \frac{3}{2} \Rightarrow H \left(\frac{7}{2}, \frac{3}{2} \right)$$

$$MH = \sqrt{\left(\frac{7}{2} - \frac{3}{2} \right)^2 + \left(2 - \frac{3}{2} \right)^2} = \sqrt{\frac{16}{4} + \frac{1}{4}} = \boxed{\sqrt{\frac{17}{4}}}$$

$$m_1 = -\frac{3m-2}{2-m}$$

$$m_2 = -\frac{m+1}{-2m+4}$$

$$m_1 m_2 = -1 \Rightarrow \frac{3m-2}{m-2} \times \frac{m+1}{2m-4} = -1$$

$$\Rightarrow 3m^2 + m - 2 = -2m^2 + 14m - 20 \Rightarrow 5m^2 - 13m + 18 = 0 \xrightarrow{\Delta < 0} \text{حالتی که در آن هیچ جوابی ندارد}$$

$$AB: xy = -x + 3 \Rightarrow y = \frac{-x+3}{x}$$

$$BC: y = -x + 4$$

$$B \begin{cases} AB \\ BC \end{cases} \rightarrow -x + 4 = \frac{-x+3}{x} \Rightarrow x=0 \quad y=-1 \Rightarrow B(0, -1)$$

$$A \begin{cases} AB \\ AC \end{cases} \rightarrow \frac{-x+3}{x} = 2x-1 \Rightarrow -x+3 = 2x-1 \Rightarrow x=1 \quad y=1 \quad A(1, 1)$$

$$C \begin{cases} AC \\ BC \end{cases} \rightarrow 2x-1 = -x+4 \Rightarrow x = \frac{5}{3} \quad y = \frac{7}{3} \quad C\left(\frac{5}{3}, \frac{7}{3}\right)$$

$$M\left(\frac{0+1}{2}, \frac{-1+1}{2}\right) = \left(\frac{1}{2}, 0\right) \Rightarrow AM = \sqrt{\left(1-\frac{1}{2}\right)^2 + \left(1-0\right)^2} = \frac{\sqrt{5}}{2}$$

$$AH = \frac{|1+1-4|}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow \frac{AM}{AH} = \frac{\frac{\sqrt{5}}{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{10}}{4}}$$

$$A(0, y) \quad y = -\frac{1}{4}x + b$$

$$B(x, 0) \quad \sqrt{2} = \frac{b}{\sqrt{\frac{1}{16}}} \Rightarrow b = 2 \Rightarrow y = -\frac{x}{4} + 2$$

$$AB = \sqrt{1+16} = \boxed{5\sqrt{5}} \Rightarrow A(0, 2), B(10, 0)$$

$$y_1 = 0 \Rightarrow -4x = 2m \Rightarrow x = -\frac{m}{2}$$

$$y_2 = 0 \Rightarrow -mx + x + 2 = 0 \Rightarrow x = \frac{2}{m-1}$$

$$\frac{2}{m-1} + \frac{m}{2} = 0 \Rightarrow 4 + m^2 - m = 0 \Rightarrow m^2 - m + 4 = 0$$

$$\Rightarrow m^2 - m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$$

$$2 \times 2 = \boxed{4}$$

$$\text{خط عمود و گزیده از } A(2, 1)$$

$$\frac{y+1}{x} = 2 \Rightarrow x_{A'} = 2$$

$$\frac{1+y}{x} = -2 \Rightarrow y_{A'} = -3$$

$$\Rightarrow A'(2, -3)$$

$$y - (-3) = 2(x - 2) \Rightarrow$$

$$\boxed{y = 2x - 7}$$

$$y - 2 = -(x - 1) \Rightarrow y = -x + 3 \rightarrow \text{خط عمود و گزیده از } A(2, 1)$$

$$\begin{cases} y = x + 2 \\ y = -x + 3 \end{cases} \rightarrow x + 2 = -x + 3 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}, y = \frac{5}{2}$$

$$\Rightarrow B(-1, 4)$$

$$\frac{1+x}{x} = 1 \Rightarrow x = -3 \Rightarrow A'(-3, 4)$$

$$\frac{y+1}{x} = 4 \Rightarrow y = 4x - 1$$

$$\Rightarrow a = -3, b = 4 \Rightarrow 2b - a = 12 - (-3) = \boxed{15}$$