

مسئله ۱۷ - یک خط مستقیم در صفحه مختصات
(۱) را بنویسید

$$A(-2, k) \quad B(2, m) \quad m_{AB} = -\frac{1}{k}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{m-k}{2-(-2)} = \frac{m-k}{4} = -\frac{1}{k} \quad m-k = -\frac{4}{k} \quad \boxed{m = k - \frac{4}{k}}$$

$$d = \sqrt{(m-k)^2 + (\Delta y)^2} = \sqrt{(k - \frac{4}{k} - k)^2 + (k - \frac{4}{k} - k)^2} = \sqrt{\frac{16}{k^2} + \frac{16}{k^2}} = \sqrt{\frac{32}{k^2}} = \frac{4\sqrt{2}}{k}$$

$$S = d^2 = \left(\frac{4\sqrt{2}}{k}\right)^2 = \frac{32}{k^2}$$

$$A(-1, 2) \quad B(1, 1) \quad C(m, y) \quad D(-1-x, y+1)$$

(۲) را بنویسید

$$AB = \sqrt{(m-(-1))^2 + (y-2)^2} = \sqrt{(m+1)^2 + (y-2)^2} = \sqrt{1+4} = \sqrt{5} = \omega \quad m_{AB} = \frac{\Delta y}{\Delta x} = \frac{1-2}{1-(-1)} = -\frac{1}{2}$$

$$m_{AB} \times m_{CD} = -1 \Rightarrow m_{CD} = \frac{2}{1} \Rightarrow \frac{\Delta y}{\Delta x} = \frac{(y+1)-y}{(-1-x)-(-1)} = \frac{1}{-x} = \frac{2}{1} \Rightarrow (-x-1) = 2 \times 1 \Rightarrow -x-1=2 \Rightarrow -x=3 \Rightarrow x=-3$$

$$DC = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-3-(-1))^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5} = \omega$$

$$\Rightarrow DC = \sqrt{\frac{11}{16} + \frac{144}{16}} = \sqrt{\frac{155}{16}} = \frac{\sqrt{155}}{4}$$

$$d = \sqrt{AB^2 + DC^2} = \sqrt{5 + \frac{155}{16}} = \sqrt{\frac{125}{16}} = \frac{5\sqrt{5}}{4}$$

(۳) را بنویسید

$$2mx + (m^2-1)y = 1 \Rightarrow (m^2-1)y = -2mx + 1 \Rightarrow y = \frac{-2m}{m^2-1}x + \frac{1}{m^2-1}$$

$$|a m_0| = \sqrt{1} \Rightarrow \frac{-2m}{m^2-1} = \sqrt{1} \rightarrow \sqrt{1} m^2 - \sqrt{1} = -2m \rightarrow \sqrt{1} m^2 + 2m - \sqrt{1} = 0$$

$$\rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(2)^2 - 4(1)(-1)}}{|1|} = \frac{\sqrt{4+4}}{1} = \frac{\sqrt{8}}{1} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

$$A(1, 9) \quad B(3, 4) \quad C(5, 11)$$

(۴) را بنویسید

$$m_{BC} = \frac{\Delta y}{\Delta x} = \frac{11-4}{5-3} = \frac{7}{2} \quad 2(x-3) = (y-4) \Rightarrow 2x-6 = y-4 \Rightarrow 2x-y-2=0$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|2(1) - 9 - 2|}{\sqrt{2^2 + (-1)^2}} = \frac{|-9|}{\sqrt{5}} = \frac{9}{\sqrt{5}} = \frac{9\sqrt{5}}{5}$$

سوال ٦

$$AB: y + 2x = 5$$

$$AC: 3y - 2x = 12$$

$$BC: 2y - 5x = -19$$

$$\begin{cases} y + 2x = 5 \\ 3y - 2x = 12 \end{cases} \xrightarrow{x(1)} \begin{aligned} &-5y - 4x = -10 \\ &+ 3y - 2x = 12 \\ \hline &-2y - 6x = 2 \end{aligned} \rightarrow y + 2(1) = 5 \rightarrow y = 3$$

$A(1, 3)$

$$\begin{cases} y + 2x = 5 \\ 2y - 5x = -19 \end{cases} \xrightarrow{x(2)} \begin{aligned} &-2y - 4x = -10 \\ &+ 2y - 5x = -19 \\ \hline &-9x = -29 \end{aligned} \rightarrow x = \frac{29}{9}$$

$B(\frac{29}{9}, 1)$

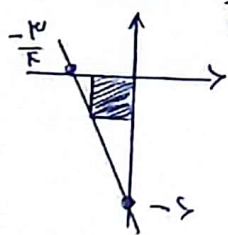
$$\begin{cases} 3y - 2x = 12 \\ 2y - 5x = -19 \end{cases} \xrightarrow{x(3)} \begin{aligned} &3y - 2x = 12 \\ &-2y + 10x = 38 \\ \hline &5y - 12x = 50 \end{aligned} \rightarrow 3y - 2(\frac{50}{5}) = 12 \rightarrow 3y = 22 \rightarrow y = \frac{22}{3}$$

$C(\frac{22}{3}, 1)$

$$m_{AC} = \frac{\Delta y}{\Delta x} = \frac{1 - 3}{\frac{22}{3} - 1} = \frac{2}{19}$$

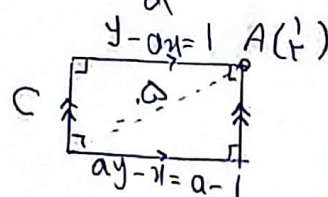
$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|\frac{2}{19}(1) + \frac{1}{19} - 1|}{\sqrt{(\frac{2}{19})^2 + 1}} = \frac{|\frac{2}{19} + \frac{1}{19} - \frac{19}{19}|}{\sqrt{\frac{4}{361} + \frac{361}{361}}} = \frac{|\frac{2+1-19}{19}|}{\sqrt{\frac{365}{361}}} = \frac{\frac{16}{19}}{\frac{\sqrt{365}}{19}} = \frac{16}{\sqrt{365}}$$

$$m = \frac{\Delta y}{\Delta x} = -\frac{4}{19} = -\frac{4}{19} \Rightarrow y = -\frac{4}{19}x - 4$$



$$\begin{aligned} -\frac{4}{19}x - 4 &= x \\ -\frac{4}{19}x - x &= 4 \\ -\frac{23}{19}x &= 4 \\ x &= -\frac{76}{23} \end{aligned}$$

سوال ٩



سوال ٧

$$y - ax = 1$$

$$ay - x = a - 1$$

$$y = ax + 1$$

$$y = \frac{x}{a} + \frac{a-1}{a}$$

$$y = x + 1$$

$$y = x$$

مربع متساوي الساقين

$$\frac{1}{1} = \frac{1}{a}$$

$$\frac{1}{a-1} \neq \frac{1}{a}$$

$$\frac{1}{1} = \frac{a}{a-1}$$

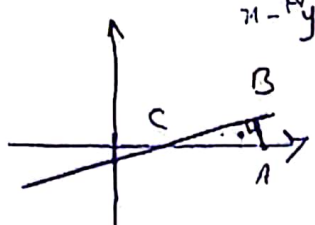
لكن النتيجة

$$A(1, 1) \rightarrow (1) - (1) + 1 = 1 \Rightarrow \text{نقطة } (1, 1) \text{ تقع على الخط}$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|1 - 1|}{\sqrt{1^2 + 1^2}} = \frac{0}{\sqrt{2}} = 0$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + a^2 = \omega^2 \Rightarrow a^2 = \frac{\omega^2}{2} - \frac{1}{2} = \frac{\omega^2 - 1}{2} \Rightarrow a = \frac{\sqrt{\omega^2 - 1}}{\sqrt{2}}$$

$$S = \frac{1}{\sqrt{2}} \times \frac{\sqrt{\omega^2 - 1}}{\sqrt{2}} = \frac{\sqrt{\omega^2 - 1}}{2}$$

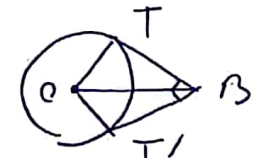
$x - 3y - 1 = 0 \Rightarrow y = \frac{1}{3}x - \frac{1}{3}$
 (سوال ۸)
 مساحت دایره ادا در صورتی است که رأس B در محل تقاطع خط با محور x باشد.


$$AB = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|1 \cdot 1 - 1|}{\sqrt{1+0}} = \frac{2}{\sqrt{1}}$$

$\Rightarrow m_{AB} = -\frac{1}{3} \rightarrow -\frac{1}{3}(x-1) = y-0 \rightarrow -\frac{1}{3}x + \frac{1}{3} = y$
 فاصله B از محور x
 $-\frac{1}{3}x + \frac{1}{3} = \frac{1}{3}x - \frac{1}{3} \quad \frac{2}{3} = \frac{2}{3}x \Rightarrow x = 1 \Rightarrow y = 0/9 \Rightarrow$
 تقاطع محور x
 $y = \frac{1}{3}x - \frac{1}{3} \Rightarrow \boxed{x=1, y=0}$

$$S = \frac{0.9 \times 3}{2} = 1.35$$

(سوال ۹) $(-\frac{1}{3}, b), (-\frac{1}{3}, a) \quad \sqrt{3} = \frac{a-b}{(-\frac{1}{3}) - (-\frac{1}{3})} \Rightarrow \frac{a-b}{-\frac{1}{3}} = \sqrt{3} \Rightarrow b-a = \frac{\sqrt{3}}{3}$
 $a = \sqrt{x^2 + y^2} = \sqrt{(-\frac{1}{3})^2 + (b-a)^2} = \sqrt{\frac{1}{9} + \frac{3}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$
 طول قطعه
 $\text{طول قطر} = \sqrt{2} a = \sqrt{2} \times \frac{2}{3} = \frac{2\sqrt{2}}{3}$

(سوال ۱۰)

 $m(OT) = \frac{0 - (-1)}{0 - (-3)} = \frac{1}{3}$
 $m(OT) \cdot m(OT') = -1 \Rightarrow m_{OT'} = -\frac{3}{1}$
 $OT = OT' = r = 2 \quad m(OT') = -\frac{3}{1} \Rightarrow y = -\frac{3}{1}x + b \xrightarrow{(0,2)} b=2 \Rightarrow y = -\frac{3}{1}x + 2$
 $T'(\frac{2}{3}, -\frac{1}{3})$
 ①, ② $OT' = \sqrt{x^2 + y^2} = \frac{2}{3} \Rightarrow \frac{2}{3}x = 2 \Rightarrow x = -\frac{3}{1}$

$T'(-3, 1) \Rightarrow \text{قطر متصف} \Rightarrow x_T + x_{T'} = x_C + x_B \Rightarrow -3 = 0 + x_B \Rightarrow x_B = -3$
 $T(-3, -1)$
 $y_T + y_{T'} = y_C + y_B \Rightarrow -1 = 0 + y_B \Rightarrow y_B = -1$
 $x_B \times y_B = -3 \times -1 = 3$