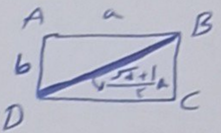
پارسا جهانگیری - یازدهم بیسر A - تکلیف ۱۸

۱-

$S_2 = L_2 \times W$
$S_1 = L_1 \times W$ $\Rightarrow \dfrac{S_2}{S_1} = \dfrac{L_2 \times W}{L_1 \times W} = \dfrac{\frac{1+\sqrt5}{2}}{\frac{5}{4}} = \dfrac{4+4\sqrt5}{10} = \boxed{\dfrac{2+2\sqrt5}{5}}$

۲-



$a^2+b^2 = \dfrac{6+2\sqrt5}{4}a^2 \Rightarrow b^2 = \dfrac{2+2\sqrt5}{4}a^2 \Rightarrow \dfrac{a^2}{b^2} = \dfrac{4}{2+2\sqrt5} = \dfrac{2}{\sqrt5+1}$

$= \dfrac{2\sqrt5-2}{4} = \boxed{\dfrac{\sqrt5-1}{2}}$

۳-

$3a-2 = -\sqrt{2a^2+4a} \overset{()^2}{\Rightarrow} 9a^2-12a+4 = 2a^2+4a \Rightarrow 7a^2-16a+4=0$

$\Rightarrow a^2-16a+28=0 \Rightarrow (a-14)(a-2)=0 \rightarrow a=\frac{14}{7}$ یا $a=\frac{2}{7}$

$a=2 \Rightarrow 4=-4$ غ ق ق

$a=\frac{2}{7} \Rightarrow -\frac{8}{7} = -\frac{8}{7}$ ✓ ق ق $\Rightarrow \dfrac{a+1}{a} = \dfrac{\frac{9}{7}}{\frac{2}{7}} = \frac{9}{2} = \boxed{4/5}$

۴-

$x \geq -1$, $x \geq 1$

$\dfrac{\sqrt{x+1}}{\sqrt{x-1}+3} - \dfrac{\sqrt{x+1}}{3-\sqrt{x-1}} = \dfrac{x-1}{\sqrt{x-1}} \Rightarrow (\sqrt{x+1})\left(\dfrac{1}{\sqrt{x-1}+3} - \dfrac{1}{3-\sqrt{x-1}}\right) = \dfrac{(\sqrt{x-1})(\sqrt{x-1})}{\sqrt{x-1}}$

$x+1$, $x>1$

$\sqrt{x-1} \neq 0$: $\sqrt{x+1}\left(\dfrac{1}{\sqrt{x-1}+3} - \dfrac{1}{3-\sqrt{x-1}}\right) = \sqrt{x-1} \Rightarrow \sqrt{x+1}\left(\dfrac{3-\sqrt{x-1}-\sqrt{x-1}-3}{9-x+1}\right) \Rightarrow \sqrt{x+1}\left(\dfrac{-2\sqrt{x-1}}{10-x}\right) = \sqrt{x-1}$

$\ominus \Rightarrow x>10$

$\Rightarrow -2\sqrt{x+1} = 10-x \overset{()^2}{\Rightarrow} 4x+4 = x^2-20x+100 \Rightarrow x^2-24x+96=0$ $x>10$ (I)

$\Rightarrow x = \dfrac{24 \pm \sqrt{192}}{2} \overset{(I)}{\Rightarrow} x = 12+4\sqrt3 \longrightarrow$ معادله ۱ ریشه⊕ دارد

۵-

$\dfrac{1}{\sqrt{2-x}+2} + \dfrac{1}{\sqrt{2-x}-2} = \dfrac{2-x}{5\sqrt{2-x}} \overset{t=\sqrt{2-x}}{\Rightarrow} \dfrac{1}{t+2} + \dfrac{1}{t-2} = \dfrac{t^2}{5t}$

$\overset{t\neq0}{\Rightarrow} \dfrac{t-2+t+2}{t^2-4} = \dfrac{t}{5} \Rightarrow \dfrac{2t}{t^2-4} = \dfrac{t}{5} \overset{t\neq0}{\Rightarrow} \dfrac{2}{t^2-4} = \dfrac{1}{5} \Rightarrow t^2-4=10$

$\Rightarrow t^2=14 \Rightarrow t=\pm\sqrt{14} \underset{t>0}{\Rightarrow} t=\sqrt{14}=\sqrt{2-x} \Rightarrow \boxed{x=-12} \overset{x>0}{\Rightarrow}$ معادله هیچ ریشه مثبتی ندارد

۶-

$\dfrac{1}{x^2} + \dfrac{1}{(1-x)^2} = \dfrac{160}{9} \Rightarrow \dfrac{(1-x)^2+x^2}{x^2(1-x)^2} = \dfrac{160}{9} \Rightarrow \dfrac{2x^2-2x+1}{x^2(x-1)^2} = \dfrac{160}{9}$

$(1-x)^2=(x-1)^2$

$\Rightarrow \dfrac{2(x^2-x)+1}{(x^2-x)^2} = \dfrac{160}{9} \overset{x^2-x=t}{\Rightarrow} \dfrac{2t+1}{t^2} = \dfrac{160}{9} \Rightarrow \dfrac{t^2+2t+1}{t^2} = \dfrac{169}{9}$

$\Rightarrow \left(\dfrac{t+1}{t}\right)^2 = \left(\dfrac{13}{3}\right)^2 \Rightarrow \begin{cases} \dfrac{t+1}{t} = \dfrac{13}{3} \Rightarrow \dfrac{1}{t} = \dfrac{10}{3} \Rightarrow t=\dfrac{3}{10} \quad (I) \\ \text{یا} \\ \dfrac{t+1}{t} = \dfrac{-13}{3} \Rightarrow \dfrac{1}{t} = \dfrac{-16}{3} \Rightarrow t = -\dfrac{3}{16} \quad (II) \end{cases}$

(I) $x^2-x=\frac{3}{10} \Rightarrow x^2-x-\frac{3}{10}=0 \overset{\Delta>0}{\Rightarrow} S=\dfrac{-b}{a}=1$

(II) $x^2-x=-\frac{3}{16} \Rightarrow x^2-x+\frac{3}{16}=0 \overset{\Delta>0}{\Rightarrow} S=\dfrac{-b}{a}=1$

(واضح است که این دو معادله جواب مشترک ندارند: $\dfrac{1\pm\sqrt{\frac{22}{5}}}{2} \neq \dfrac{1\pm\sqrt{\frac{1}{4}}}{2}$)

مجموع ریشه‌ها معادله $= 1+1 = \boxed{2}$

پاسخ جمع‌بندی - یازدهم تجربی A - تکلیف ۱۸

۷- $\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2+\sqrt{-x^2+6x-8}}=x+2$

$-x^3+4x^2+25x-100\geq 0$

$x^2(4-x)-25(4-x)\geq 0$

$(x^2-25)(4-x)\geq 0 \Rightarrow (x-5)(x+5)(4-x)\geq 0$

$\longrightarrow$ ریشه‌ها: $-5,\ 4,\ 5$ ؛ علامت: $+\ -\ +\ -$

$(-\infty,-5]\cup[4,5]$ (I)

$-x^2+6x-8\geq 0$

$\Rightarrow x^2-6x+8\leq 0$

$(x-4)(x-2)\leq 0$

ریشه‌ها: $2,\ 4$ ؛ علامت: $+\ -\ +$

$[2,4]$ (II)

$(I)\cap(II) \Rightarrow \{4\}$

$x=4 \Rightarrow \sqrt{4}+\sqrt{16}=4+2$

$\Rightarrow 2+4=6$ ق ق

معادله فقط یک جواب دارد $(x=4)$

۸- $y=|x+2|+|x-1| \longrightarrow \begin{cases} y=2x+1 & x\geq 1\\ y=3 & -2\leq x\leq 1\\ y=-2x-1 & x\leq -2\end{cases}$

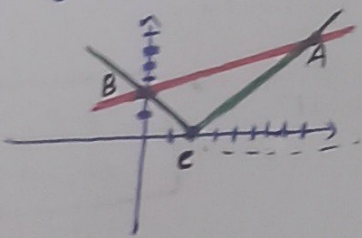
$3y+x=17 \Rightarrow y=-\frac{1}{3}x+\frac{17}{3}$

$\begin{cases} x\geq 1 \Rightarrow 2x+1=-\frac{1}{3}x+\frac{17}{3} \Rightarrow 6x+3=-x+17 \Rightarrow 7x=14 \Rightarrow x=2 \overset{x\geq 1}{\Rightarrow} \text{ق ق} \quad \xrightarrow{y=2x+1} y=5\\ -2\leq x\leq 1 \Rightarrow 3=-\frac{1}{3}x+\frac{17}{3} \Rightarrow -x+17=9 \Rightarrow x=8 \overset{-2\leq x\leq 1}{\Rightarrow} \text{غ ق ق}\\ x\leq -2 \Rightarrow -2x-1=-\frac{1}{3}x+\frac{17}{3} \Rightarrow -6x-3=-x+17 \Rightarrow -5x=20 \Rightarrow x=-4 \overset{x\leq -2}{\Rightarrow} \text{ق ق} \quad \xrightarrow{y=-2x-1} y=7\end{cases}$

$\Rightarrow A\begin{vmatrix}2\\5\end{vmatrix},\ B\begin{vmatrix}-4\\7\end{vmatrix} \Rightarrow AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=\sqrt{(6)^2+(-2)^2}=\sqrt{40}=\boxed{2\sqrt{10}}$

۹- $y=\sqrt{x^2-4x+4}=\sqrt{(x-2)^2}=|x-2| \longrightarrow \begin{cases} x-2 & x\geq 2\\ -x+2 & x\leq 2\end{cases}$

$y=\frac{1}{2}x+2$



$C\begin{vmatrix}2\\0\end{vmatrix}$

$\begin{cases} x\geq 2 \Rightarrow x-2=\frac{1}{2}x+2 \Rightarrow \frac{1}{2}x=4 \Rightarrow x=8 \overset{x\geq 2}{\Rightarrow} \text{ق ق} \quad \xrightarrow{y=x-2} y=6\\ x\leq 2 \Rightarrow -x+2=\frac{1}{2}x+2 \Rightarrow -x=\frac{1}{2}x \Rightarrow x=0 \overset{x\leq 2}{\Rightarrow} \text{ق ق} \quad \xrightarrow{y=-x+2} y=2\end{cases}$

$A\begin{vmatrix}8\\6\end{vmatrix} \quad B\begin{vmatrix}0\\2\end{vmatrix}$

$BC: y=-x+2 \Rightarrow y+x-2=0$

$\overline{AH}=\frac{|6+8-2|}{\sqrt{2}}=\frac{12}{\sqrt{2}}=6\sqrt{2}$

$\overline{BC}=\sqrt{(2-0)^2+(0-2)^2}=\sqrt{8}=2\sqrt{2}$

$\Rightarrow S=\frac{\overline{AH}\times\overline{BC}}{2}=\frac{6\sqrt{2}\times 2\sqrt{2}}{2}=\boxed{12}$

۱۰- $\frac{1}{x}+\frac{1}{x+9}=\frac{1}{20} \Rightarrow \frac{2x+9}{x^2+9x}=\frac{1}{20} \Rightarrow x^2+9x=40x+180$

$\Rightarrow x^2-31x-180=0 \Rightarrow (x-36)(x+5)=0 \Rightarrow \boxed{x=36}$ یا $x=-5$ غ ق ق

بهروز به تنهایی در ۳۶ ساعت و

فرهاد به تنهایی در ۴۵ ساعت یک مجله را تایپ می‌کند.