۱۴ شمارهٔ ۱۹ یازدهم پسر A یاسین ولیسی زاده

① ۲

$\alpha, \beta \quad \alpha = 3\beta$

$P = \alpha\beta = \frac{4}{3} \Rightarrow 3\beta \times \beta = \frac{4}{3} \Rightarrow 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9}$ $\begin{cases} \beta = \frac{2}{3} \Rightarrow \alpha = 2 \Rightarrow S = \frac{8}{3} = \frac{a}{3} \Rightarrow a = 8 \\ \beta = \frac{-2}{3} \Rightarrow \alpha = -2 \Rightarrow S = \frac{-8}{3} = \frac{a}{3} \Rightarrow a = -8 \end{cases}$ $\left(-\frac{b}{a}\right)$

$\Rightarrow |8 - (-8)| = 16$ ✓

② ۲

$\alpha, \beta \Rightarrow \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} \rightsquigarrow \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \Rightarrow \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \Rightarrow \frac{\sqrt{S + 2\sqrt{P}}}{\sqrt{P}} = 5 \Rightarrow \sqrt{S + 2\sqrt{P}} = 5\sqrt{P}$

$\Rightarrow S + 2\sqrt{P} = 25P \Rightarrow S = 25\left(\frac{1}{36}\right) - 2\left(\frac{1}{6}\right) = \frac{13}{36} \Rightarrow \frac{m+14}{36} = \frac{13}{36} \Rightarrow m = -1$

$S = \frac{m+14}{36}$ $\Rightarrow -x^2 + 3x + 2 = 0$ حاصلضرب ریشه‌ها $P = \frac{c}{a} = -2$ ✓

$P = \frac{1}{36}$

③ ۲

$a + \beta = 1 \Rightarrow \beta = 1 - a \Rightarrow a\beta = a(1-a) = -2 \Rightarrow a^2 - a - 2 = 0 \rightsquigarrow \begin{cases} a = 2 \\ \beta = -1 \end{cases}$ یا برعکس

$\Rightarrow -4 + k + 9 - 2 = 0 \Rightarrow k = -3$ ✓

④ ۰

$S = -6 \quad P = a$

$\alpha, \beta = \frac{-6 \mp \sqrt{36 - 4a}}{2} \xrightarrow{\alpha < \beta} \alpha = -3 - \sqrt{9 - a}$

$\alpha^3 + 2(\alpha^2 + \beta^2) = \alpha^3 + 2(S^2 - 2P)$

$(-3 - \sqrt{9-a})^3 + 2(36 - 2a) = 12\sqrt{2} + 85 \rightarrow a = 1$

⑤ ۲

$t = x^2 \rightsquigarrow t^2 - 7t - 5 = 0 \Rightarrow$ $\begin{cases} t_1 < 0 \xrightarrow{t = x^2 \geq 0 \text{ است}} \text{غ ق ق} \\ t_2 > 0 \Rightarrow t_2 \nearrow^{x^2}_{(-x)^2} \end{cases}$ $\Rightarrow -x, x$ را دارد ریشه‌ی $\rightarrow S = x - x = 0$

$S > 0$

$P < 0$

$\Rightarrow t_2 = \frac{7 + \sqrt{69}}{2}$

$\Rightarrow 2P^2 - 3SP + 2S = 2P^2 = \frac{118 + 14\sqrt{69}}{2}$ ✓ $\Rightarrow P = x(-x) = -x^2 = -t_2$

⑥ ۲

$w, r \rightsquigarrow w' + \frac{1}{2}, r' + \frac{1}{2} \rightsquigarrow S = \frac{a}{2} = \frac{2r' + 2w' + 2}{2} \Rightarrow a = 2r' + 2w' + 2 \Rightarrow a = 1$

$\rightarrow P = \frac{b}{2} = \frac{4r'w' + 2(r' + w') + 1}{4}$

$w', r' \rightsquigarrow S = w' + r' = -\frac{1}{2} \Rightarrow 2w' + 2r' = -1$

$\rightarrow P = \frac{-6}{2} = -3 \Rightarrow w'r' = -3$

$\Rightarrow b = \frac{4(-3) + 2(-\frac{1}{2}) + 1}{2}$

$\Rightarrow b = -6$

$\Rightarrow \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = -2$ ✓

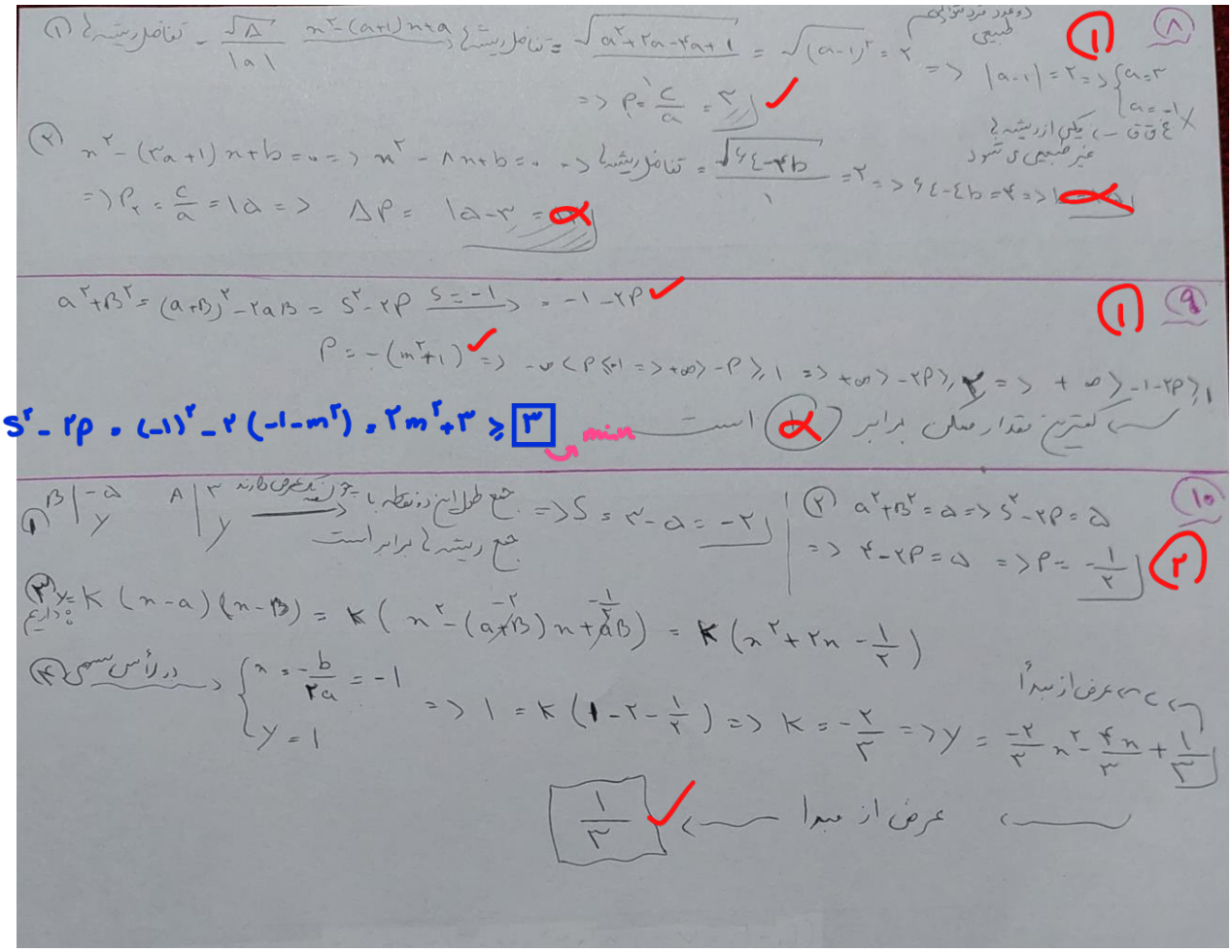
⑦ ۰

$S = \alpha + \beta = 1 \rightarrow \alpha = 1 - \beta$

$40\beta^3 + 20(1-\beta)^3 - 20\beta = 17 \rightarrow 20\beta^2 - 20\beta + 1 = 0$

$20\beta(\beta - 1) = -1 \rightarrow -20\alpha\beta = -1 \rightarrow \alpha\beta = \frac{1}{20} = -\frac{b}{a} \rightarrow a = -20b$

$-20bx^2 + 20bx - b = 0 \rightarrow |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \boxed{\frac{2}{\sqrt{5}}}$



$\alpha^2+\beta^2 = S^2-2P = (-1)^2-2(-1-m^2)=2m^2+3 \geq 3$ ← min

است α کمترین مقدار ممکن برابر

ادامه ۸

$x^2-(3a+1)x+b=0 \xrightarrow{a=3} x^2-10x+b=0$

$S=\alpha+\alpha+2=10 \rightarrow \alpha=4$

ریشه‌ها α و $\alpha+2$ ⇐

دو عدد زوج متوالی

$P=4(6)=24$

اختلاف $=24-3=\boxed{21}$