باسمه تعالی

نام و نام خانوادگی: علی ربانی

پاسخنامه تکلیف شماره ۱۹

کلاس یازدهم A

۲۰ آفرین!

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$3x^2-ax+4=0$ ریشه $= x_1, 3x_1$

$27x^2-3ax+4=0$

$3x^2-12x^2+4=0$

$-9x^2+4=0$

$27x^2-3ax+4 = 3x^2-ax+4$

$9x^2=4 \Rightarrow x^2=\frac{4}{9} \Rightarrow x=\pm\frac{2}{3}$

$24x^2-2ax=0 \quad 12x^2-ax=0 \Rightarrow ax=12x^2$

$x=\frac{2}{3} \rightarrow \frac{4}{3}-a\frac{2}{3}+4=0 \qquad a\frac{2}{3}=\frac{4}{3}+\frac{12}{3}=\frac{16}{3} \Rightarrow a_1=8$ ✓

$x=-\frac{2}{3} \rightarrow \frac{4}{3}+\frac{2}{3}a+4=0 \qquad -\frac{2}{3}a=\frac{16}{3} \Rightarrow a_2=-8$ ✓ $\quad a_1-a_2=8--8=16$ ✓

(۲)

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$\sqrt{\frac{1}{x_1}}+\sqrt{\frac{1}{x_2}}=5 \qquad 36x^2-(m+14)x+1=0$

$\frac{m+14}{36}=\frac{13}{36} \Rightarrow m=-1$

$\Rightarrow \sqrt{\frac{1}{\frac{1}{4}}}+\sqrt{\frac{1}{\frac{1}{9}}}=5 \qquad x_1=\frac{1}{4} \quad x_2=\frac{1}{9}$

$mx^2+4x+p=0 \qquad -x^2+4x+p=0 \qquad P=-2$ ✓

(۲)

۳-

$4x^3+kx^2-9x-2=0 \qquad \alpha+\beta=1 \qquad \alpha\beta=-2$

$\beta=-\alpha+1 \qquad \alpha\beta=-2=(-\alpha+1)(\alpha)=-2$

$\Rightarrow \alpha=2 \quad \beta=-1$

$\alpha=-1 \quad \beta=2$

$(\alpha-2)(\alpha+1)=0 \Leftarrow -\alpha^2+\alpha+2=0$

$\alpha^2-\alpha-2=0$

$4(8)+4k-9(2)-2 = 32+4k-20=0$

$4k=-12 \Rightarrow k=-3$ ✓

(۲)

۴-

$x^2+6x+a=0 \qquad \alpha<\beta<0 \qquad 2\alpha^2+3\beta^2=12\sqrt{2}+85$

(۱)

$S=-6 \Rightarrow \alpha+\beta=-6 \Rightarrow \beta=-\alpha-6 \qquad 2\alpha^2+3(-\alpha-6)^2=12\sqrt{2}+85$

$\alpha^2+12\alpha+36$

$\Delta=\underset{576}{24^2}+260+240\sqrt{2}=836+240\sqrt{2} \qquad 2\alpha^2+3\alpha^2+24\alpha+72=12\sqrt{2}+85$

$\alpha=\frac{-24\pm\sqrt{836+240\sqrt{2}}}{10} \leftarrow 5\alpha^2+24\alpha-13-12\sqrt{2}$

$\sqrt{\underset{(20\sqrt{2})^2}{800}+\underset{6^2}{36}+240\sqrt{2}}=\sqrt{(20\sqrt{2}+6)^2}=20\sqrt{2}+6$

$\alpha<0 \Downarrow$

$\alpha=\frac{-24-\sqrt{836+240\sqrt{2}}}{10}=\frac{-24-6-20\sqrt{2}}{10}=\frac{-30-20\sqrt{2}}{10}=-3-2\sqrt{2}$

$\alpha+\beta=-6 \qquad -3-2\sqrt{2}+\beta=-6 \qquad \beta=-3+2\sqrt{2}$

$a=\alpha\beta=(-3-2\sqrt{2})(-3+2\sqrt{2})=9-8=1$ ✓ جواب نهایی

(۲)

۵- (۲)

$x^4 - 7x^2 - 5 = 0 \xrightarrow{x^2 = t} t^2 - 7t - 5 = 0$

$t = \frac{7 \pm \sqrt{69}}{2} \Rightarrow x^2 = \frac{7 + \sqrt{69}}{2}$, $x = \begin{cases} +\sqrt{\frac{7+\sqrt{69}}{2}} \\ -\sqrt{\frac{7+\sqrt{69}}{2}} \end{cases} \Rightarrow S = 0$

$P = \sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}} = -\sqrt{\frac{(7+\sqrt{69})^2}{4}} = \frac{-7-\sqrt{69}}{2}$

$2P^2 - 2SP + 4S = 2P^2 = 2 \times \left(\frac{-7-\sqrt{69}}{2}\right)^2 = \frac{49 + 69 + 14\sqrt{69}}{2} = \frac{118 + 14\sqrt{69}}{2}$

$= \boxed{59 + 7\sqrt{69}}$ ✓

۶- (۲)

① $2ax^2 + ax - 6 = 0$ $\quad 2x^2 + x - 6$

② $2x^2 - ax + b = 0$

$S_① = \frac{-a}{2a} = -\frac{1}{2} = \alpha + \beta$

$P_① = \frac{-6}{2a} = -\frac{3}{a} = \alpha\beta \xrightarrow{a=1} \alpha\beta = -3$

$S_② = \alpha + \beta + 1 = \frac{a}{2} \Rightarrow \boxed{a = 1}$

$P_② = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \underbrace{\alpha\beta}_{-3} + \frac{1}{2}\underbrace{(\alpha+\beta)}_{-\frac{1}{2}} + \frac{1}{4} =$

$-3 - \frac{1}{4} + \frac{1}{4} = -3$

$P_2 = \frac{b}{2} = -3 \Rightarrow \boxed{b = -6}$

$\left[\frac{-6 \times 1}{4}\right] = \left[-\frac{3}{2}\right] = \boxed{-2}$ ✓ [illegible]

۷- (۲)

$20\beta^3 + 20\beta^2 + 20\alpha^2 - 20\beta = 17$ $\qquad ax^2 - ax - b = 0$

$20(\beta^3 - \beta + \beta^2 + \alpha^2) = 17$ $\qquad \alpha + \beta = \frac{a}{a} = 1 \Rightarrow \beta - 1 = -\alpha$

$\beta^3 - \beta + \beta^2 + \alpha^2 = \frac{17}{20}$

$\beta(\beta - 1) + S^2 - 2P = \frac{17}{20}$, $\quad \beta(\beta-1) = -\alpha\beta = \frac{b}{a}$, $\quad -2P = \frac{2b}{a}$

$S^2 + \frac{2b}{a} = \frac{17}{20}$

$\frac{20}{20} + \frac{3b}{a} = \frac{17}{20} \Rightarrow \frac{3b}{a} = -\frac{3}{20} \Rightarrow \boxed{\frac{b}{a} = -\frac{1}{20}}$

اختلاف ریشه‌ها $= \frac{\sqrt{\Delta}}{a} = \frac{\sqrt{a^2 + 4ab}}{a} = \sqrt{\frac{a^2 + 4ab}{a^2}} = \sqrt{1 + \frac{4b}{a}}$

$\sqrt{1 - \frac{4}{20}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ ✓ $\quad \left(1 - \frac{1}{5}\right)$

$\boxed{\frac{2\sqrt{5}}{5}}$

۸- (۲)

$x^2 - (a+1)x + a = 0$ $\quad x , x+2$

$S: 2x + 2 = a + 1$ $\quad 2x + 1 = a$

$x(x+2) = a$ $\quad x^2 + 2x = a$ جواب‌ها طبیعی

$x^2 + 2x = 2x + 1$ $\quad x^2 = 1$ $\quad x = \pm 1 \to +1$

$x = 1$, $x + 2 = 3 \Rightarrow \boxed{a = 3}$

$x^2 - (3a+1)x + b = 0$ $\quad y , y+2$

$x^2 - 10x + b$

$2y + 2 = 10$ $\quad 2y = 8$ $\quad y = 4$

$y = 4$, $y + 2 = 6$

$6 \times 4 = \boxed{24}$ ✓ $\quad 24 - 3 = \boxed{21}$ ✓

$2 \times 1 = 2$ ✓

۹ - ۳

$x^2 + x - 1 - m^2 = 0$

$S = \alpha + \beta = 1$ $\quad$ $\min \alpha^2 + \beta^2 = S^2 - 2P$

$P = \alpha\beta = -(1 + m^2)$ $\quad$ $1 - 2P$

$1 - -(2 + 2m^2) = 2m^2 + 3$

ج $2m^2 + 3$ (همواره مثبت) کمترین مقدار → $m = 0 \Rightarrow 2(0)^2 + 3 = 3$ ✓

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۱۰ - $\frac{1}{2}$

$\begin{cases} -1 \\ 1 \end{cases}$ → AB شیب $= \frac{2-0}{2} = -1$

$y = ax^2 + bx + c \Rightarrow 1 = a - b + c$

$-\frac{b}{2a} = -1$ $\quad$ $2a = b$ $\quad$ $ax^2 + 2ax + c$

$-\frac{\Delta}{4a} = 1 = \frac{4ac - b^2}{4a}$ $\quad 4a^2 \quad$ $= \frac{4a(c-a)}{4a} = 1$

$c - a = 1$

$\Rightarrow c = a + 1$

$y = ax^2 + 2ax + a + 1$

$\alpha^2 + \beta^2 = 5 \Rightarrow S^2 - 2P = 5 \Rightarrow 4 - \frac{2a+2}{a} = 5$

$-\frac{2a+2}{a} = 1$ $\quad$ $2a + 2 = -a$

$3a = -2$

$a = -\frac{2}{3}$

$y = -\frac{2}{3}x^2 - \frac{4}{3}x + \frac{1}{3}$ ✓ → عرض از مبدأ

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