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$4\alpha = \frac{a}{3}$

$3\alpha^2 = \frac{4}{3} \quad \alpha = \pm\frac{2}{3} \Rightarrow \begin{cases} \alpha = \frac{2}{3} \Rightarrow a_1 = 8 \\ \alpha = -\frac{2}{3} \Rightarrow a_2 = -8 \end{cases} \Rightarrow a_1 - a_2 = 16$

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$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5 \qquad \alpha\beta = \frac{1}{25}$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{2}{\sqrt{\alpha\beta}} = 25$

$-x^2 + 5x + 2 = 0 \qquad S = -2$

$\frac{\alpha+\beta}{\alpha\beta} + \frac{2}{\sqrt{\alpha\beta}} = 25 \qquad 24(\alpha+\beta) + 12 = 25 \qquad 24\left(\frac{m+12}{24}\right) = 15 \qquad m = -1$

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$4x^3 + kx^2 - 9x - 2 = 4(x-\alpha)(x-\beta)(x-\gamma)$

$-4\alpha\beta\gamma = -2 \qquad \gamma = -\frac{2}{8} = -\frac{1}{4}$

$k = 4(-\alpha\beta - \gamma)$

$-\frac{5}{4}$

$k = -5$

۴-

$\frac{5}{2}(\alpha^2+\beta^2) + \frac{1}{2}(\alpha^2-\beta^2) = 12\sqrt{2} + 85 \qquad S = a$

$\frac{5}{2}(24 - 2a) + \frac{1}{2}(\alpha-\beta)(-4) = 12\sqrt{2} + 85 \qquad |\alpha-\beta| = \frac{\sqrt{25-4a}}{1}$

$90 - 5a + 3\sqrt{25-4a} = 12\sqrt{2} + 85 \qquad \alpha - \beta = -\sqrt{25-4a}$

$a \in \mathbb{Q} \Rightarrow 90 - 5a = 85 \Rightarrow \boxed{a = 1}$

$25 - 4a = 21 \Rightarrow a = 1$

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$t^2 - 7t - 5 = 0 \qquad x^2 < \frac{7+\sqrt{49}}{2} \qquad S = 0 \qquad P = \frac{-7-\sqrt{49}}{2}$

$\frac{7-\sqrt{49}}{2}$ صحیح

$2P^2 - 3SP + 2S = 4\left(\frac{49+49+14\sqrt{49}}{4}\right) + 0 + 0 = 98 + 7\sqrt{49}$

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$\alpha + \beta = \frac{-a}{2a} = -\frac{1}{2}$

$\alpha + \frac{1}{2} + \beta + \frac{1}{2} = \frac{1}{2} = \frac{a}{2} \Rightarrow a = 1$

$\alpha\beta = \frac{-6}{2} = -3$

$(\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha+\beta) + \frac{1}{4} = -3 + \frac{1}{2}\times(-\frac{1}{2}) + \frac{1}{4} = -3 = \frac{b}{2} \Rightarrow b = -6$

$\left[\frac{-6}{4}\right] = -2$

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$\alpha + \beta = \frac{a}{5} = 1$

$\alpha = 1 - \beta$

$|\alpha - \beta| = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$2\beta^2 + (1-\beta)^2 - \beta = \frac{17}{20}$

$3\beta^2 - 3\beta + \frac{3}{20} = 0$

$\beta^2 - \beta + \frac{1}{20} = 0 \qquad \beta = \frac{1 \pm \sqrt{1 - \frac{1}{5}}}{2}$

$\frac{1}{2} + \frac{1}{2\sqrt{5}} \qquad \alpha = \frac{1}{2} - \frac{1}{2\sqrt{5}}$

$\frac{1}{2} - \frac{1}{2\sqrt{5}} \qquad \alpha = \frac{1}{2} + \frac{1}{2\sqrt{5}}$

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$\alpha(\alpha+2)=a$

$\alpha+\alpha+2=a+1$

$\alpha^2+2\alpha-2\alpha=1$

$\alpha \swarrow 1$ غیرقابل قبول

$\alpha=1 \Rightarrow a=2$

$\alpha+2=3 \Rightarrow b\times 1=c$

$\beta+\beta+2=10$

$\beta=4$ } $4\times 4=24$

$\beta+2=6$

$24-c=21$

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$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-1)^2+2+2m^2=2m^2+3 \geq 3$

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$x_{ext}=\frac{-b}{2a}=-1$

$y_{ext}=1$

$y=a(x+1)^2+1$

$y=ax^2+2ax+a+1$

$\alpha^2+\beta^2=5$

$(\frac{-2a}{a})^2-2\times(\frac{a+1}{a})=5$

$4-2-\frac{2}{a}=5$

$-\frac{2}{a}=3 \quad a=-\frac{2}{3} \Rightarrow y=-\frac{2}{3}x^2-\frac{4}{3}x+\frac{1}{3}$

$x=0 \Rightarrow y=\frac{1}{3}$