تکلیف ۱۹ شهاب محقق

سوال ۱)

$3x^2 - ax + 4 = 0 \qquad \alpha = 3\beta \qquad P = \alpha \cdot \beta = 3\beta^2 \Rightarrow 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \to \beta = \pm\frac{2}{3}$

$S = \alpha + \beta = 4\beta = \frac{a}{3}$

$\to \beta = \frac{2}{3} \qquad 4\left(\frac{2}{3}\right) = \frac{a}{3} \Rightarrow a_1 = 8$

$\to \beta = -\frac{2}{3} \qquad 4\left(\frac{-2}{3}\right) = \frac{a}{3} \Rightarrow a_2 = -8$

$|a_1 - a_2| = |8 - (-8)| = 16$

سوال ۲)

$36x^2 - (m+14)x + 1 = 0 \qquad \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$

$\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \Rightarrow \frac{\sqrt{S+2\sqrt{P}}}{\sqrt{P}} = 5 \Rightarrow \begin{cases} S = \frac{m+14}{36} \\ P = \frac{1}{36} \end{cases}$

$\frac{\sqrt{\frac{m+14}{36} + 2\left(\frac{1}{6}\right)}}{\sqrt{\frac{1}{36}}} = 5 \qquad 6\sqrt{\frac{m+26}{36}} = 5 \qquad \sqrt{\frac{m+26}{36}} = \frac{5}{6}$

$\Rightarrow \frac{m+26}{36} = \frac{25}{36} \Rightarrow m = 25 - 26 = \boxed{-1}$

$mx^2 + 3x + 2 = 0 \qquad P = \frac{2}{m} = \frac{2}{-1} = -2$

سوال ۳)

$4x^3 + kx^2 - 9x - 2 = 0$

$\alpha + \beta = 1 \to \alpha = -\beta + 1$

$\alpha\beta = -2 \to \beta(-\beta+1) = -2$

$-\beta^2 + \beta = -2$

$\beta^2 - \beta - 2 = 0$

$(\beta - 2)(\beta + 1) = 0$

$\begin{cases} \beta = 2 \to \alpha = -1 \\ \beta = -1 \to \alpha = 2 \end{cases}$

$\xrightarrow{\text{با جایگذاری}}$ α یا β در معادله فوق

$x = \alpha$ یا β

$4(-1)^3 + k(-1)^2 - 9(-1) - 2 = 0$

$-4 + k + 9 - 2 = 0 \to k = -3$

سوال ۴)

$x^2 + 6x + a \qquad \alpha < \beta < 0 \qquad 3\alpha^2 + 2\beta^2 = 12\sqrt{32} + 85 = 3\sqrt{32} + 85$

$\alpha = \frac{-6 - \sqrt{36-2a}}{2}$

$2(\alpha^2 + \beta^2) + \alpha^2 = 12\sqrt{32} + 85 \Rightarrow 2(S^2 - 2P) + \alpha^2 = 12\sqrt{32} + 85$

$2(36 - 2a) + \left(\frac{-6 - \sqrt{36-2a}}{2}\right)^2 = 12\sqrt{32} + 85 \Rightarrow 72 - 4a + \frac{36 + 36 - 2a + 12\sqrt{36-2a}}{4} =$

$\Rightarrow 72 - 4a + \frac{72 - 2a + 12\sqrt{36-2a}}{4} = 72 - 4a + \frac{72-2a}{4} + 3\sqrt{36-2a} = 3\sqrt{32} + 85$

$\Rightarrow 36 - 2a = 32 \to a = \frac{36-32}{2} = \frac{4}{2} = \boxed{2}$

سوال ۵)

$x^4 - 7x^2 - 5 = 0 \qquad 2P^2 - 3SP + 2S = ?$

$$x^2 = \frac{7 \pm \sqrt{49+4(5)}}{2} = \frac{7 \pm \sqrt{69}}{2} \xrightarrow{x^2 \geq 0} \begin{cases} \frac{7+\sqrt{69}}{2} & \text{قق} \\ \frac{7-\sqrt{69}}{2} & \text{غق} \end{cases}$$

$$x_{1,2} = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \Rightarrow \begin{cases} S = 0 \\ P = -\frac{7+\sqrt{69}}{2} \end{cases}$$

$$2P^2 - 3SP + 2S = 2P^2 - 3(0)P + 2(0) = 2P^2 \Rightarrow = 2\left(-\frac{7+\sqrt{69}}{2}\right)^2$$

$$= 2 \times \left(\frac{49+69+14\sqrt{69}}{4}\right) = \frac{118+14\sqrt{69}}{2} = 59 + 7\sqrt{69}$$

سوال ۶)

$\left[\frac{ab}{4}\right] = ?$

$2x^2 - ax + b = 0 \qquad \alpha+0.5, \beta+0.5 \Rightarrow S = (\alpha+0.5)+(\beta+0.5) = \frac{a}{2}$

$2ax^2 + ax - 6 = 0 \qquad \alpha, \beta \Rightarrow S = \alpha+\beta = \frac{-a}{2a} = \frac{-1}{2}$

$$\alpha+\beta = \frac{a}{2} - 1 \Rightarrow \frac{-1}{2} = \frac{a}{2} - 1$$

$$\frac{1}{2} = \frac{a}{2} \Rightarrow \boxed{a = 1}$$

$$\begin{cases} P = (\alpha+0.5)(\beta+0.5) = \alpha\beta + 0.5(\alpha+\beta) + 0.25 = \frac{b}{2} \\ P = \alpha\beta = \frac{-6}{2} = -3 \end{cases} \Rightarrow -3 + 0.5\left(\frac{-1}{2}\right) + 0.25 = \frac{b}{2}$$

$$\Rightarrow \frac{b}{2} = -3 \Rightarrow \boxed{b = -6}$$

$$\left[\frac{ab}{4}\right] = \left[\frac{(1)(-6)}{4}\right] = \left[\frac{-3}{2}\right] = -2$$

سوال ۷)

$ax^2 - ax - b = 0 \qquad 40\beta^2 + 20\alpha^2 - 20\beta = 17 \Rightarrow 20\beta^2 + 20\alpha^2 + 20\beta^2 - 20\beta = 17$

$$\begin{cases} S = 1 \\ P = \frac{-b}{a} \end{cases} \qquad \begin{cases} 20\underbrace{(\beta^2+\alpha^2)} + 20\beta^2 - 20\beta = 17 \\ S^2 - 2P = (1)^2 - 2\left(\frac{-b}{a}\right) = 1 + \frac{2b}{a} \end{cases}$$

$$ax^2 - ax - b \xrightarrow{\div a} x^2 - x - \frac{b}{a} \xrightarrow{\times 20} 20x^2 - 20x - \frac{20b}{a} = 0 \xrightarrow{x=\beta}$$

$$\Rightarrow 20\beta^2 - 20\beta = \frac{20b}{a}$$

$$20\left(1 + \frac{2b}{a}\right) + \frac{20b}{a} = 17 \Rightarrow 20 + \frac{40b}{a} + \frac{20b}{a} = 17 \Rightarrow \frac{60b}{a} = -3 \Rightarrow \frac{-b}{a} = \frac{1}{20}$$

اختلاف ریشه‌ها $\Rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{1-4\left(\frac{-b}{a}\right)}}{1} = \frac{\sqrt{1-\left(\frac{1}{5}\right)}}{1} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$

سوال ۸)

$x^2-(a+1)x+a=0$ ریشه‌ها $\alpha , \alpha+2$

$S=\alpha+\alpha+2=2\alpha+2=a+1 \Rightarrow 2\alpha+1=a$

$P=\alpha(\alpha+2)=\alpha^2+2\alpha=a \Rightarrow \alpha^2+2\alpha=a$ $\Big\} \Rightarrow \alpha^2+2\alpha=2\alpha+1$

$\alpha^2=1 \to \alpha=\pm 1$

$\xrightarrow{\alpha \in N} \alpha=1$ قق

$a=2\alpha+1=2(1)+1=\boxed{3}$

$x^2-(3a+1)x+b=0$

$x^2-(3(3)+1)x+b=0 \to x^2-10x+b \to$ ریشه‌ها β و $\beta+2$

$S=\beta+(\beta+2)=2\beta+2=10 \Rightarrow \beta=4 , \beta+2=4+2=6$

$P=4\times 6=\boxed{24=b}$ اختلاف $=|b-a|=|24-3|=21$

سوال ۹)

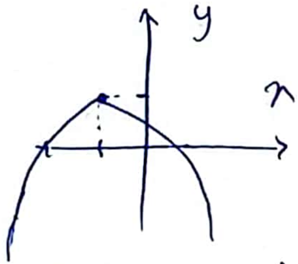
$x^2+x-1-m^2=0$ $\alpha^2+\beta^2=?$

$S^2-2P=(1)^2-2(-1-m^2)=1+2+2m^2=2m^2+3$ $y_S=-\frac{\Delta}{4a}=-\frac{0-4(2)(3)}{4(2)}=\frac{8\times 3}{8}$ $\boxed{=3}$

سوال ۱۰)

$A(3,y)$ $B(-5,y)$ $y_S=1$ $\alpha^2+\beta^2=5$

$x_S=\frac{x_A+x_B}{2}=\frac{3+(-5)}{2}=-1 \Rightarrow 2x_S=S \Rightarrow \boxed{S=-2}$



$S^2-2P=5 \Rightarrow (-2)^2-2P=5$ $4-2P=5$ $P=-\frac{1}{2}$

$a(x^2-Sx+P)=a(x^2+2x-\frac{1}{2}) \xrightarrow{S(-1,1)}$

$\begin{cases} a((-1)^2+2(-1)-\frac{1}{2})=1 \\ a(1-2-\frac{1}{2})=1 \Rightarrow a=\frac{-2}{3} \end{cases}$

عرض از مبدا $=a\times -\frac{1}{2}=\frac{-1}{2}\times\frac{-2}{3}=\boxed{\frac{1}{3}}$