نام و نام خانوادگی ...کوروش... پاسخنامه تشریحی تکلیف شماره ...19... کلاس ...نهم پسر A...

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$P = \frac{4}{\alpha^2} = \alpha^2 \quad \alpha = \frac{2}{\alpha} \quad 2\alpha = 2 \quad \alpha^2 - 2a + 2 = 0 \quad a = 8$

$\alpha = -\frac{2}{\alpha} \quad 2\alpha = -2 \quad \alpha^2 + 2a + 4 = 0 \quad a = -8$

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$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \quad \frac{\alpha+\beta+2\sqrt{\alpha\beta}}{\alpha\beta} = 25 \quad \frac{S+2\sqrt{P}}{P} = 25$

$S = \frac{m+14}{24} \quad P = \frac{1}{24} \quad \frac{\frac{m+14}{24} + \frac{1}{24}}{\frac{1}{24}} = \frac{\frac{m+14+1}{24}}{\frac{1}{24}} = m + 24 = 25 \quad m = -1$

$P' = \frac{2}{-1} = -2$

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$\alpha + \beta = 1 \quad \beta = 1 - \alpha$

$\alpha\beta = -2 \quad \alpha(1-\alpha) = -2 \rightarrow -\alpha^2 + \alpha = -2 \quad \alpha^2 - \alpha - 2 = 0 \quad (\alpha - 2)(\alpha + 1) = 0$

$\alpha = 2, \alpha = -1$

$\alpha = \alpha = -1 \rightarrow -4 + k + 9 - 2 = 0 \quad k = -3$

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$\alpha^2 + 2(\alpha^2 + \beta^2) = \alpha^2 + 2(S^2 - 2P) = 18a + 12\sqrt{2} \quad S = -4 \quad P = a$

$\Delta = 24 - 2a \quad \alpha = \frac{-4 - \sqrt{24 - 2a}}{2} \Rightarrow \alpha^2 = \frac{24 + 8 - 2a + 12\sqrt{24 - 2a}}{4} = 18 - a + 4\sqrt{9 - a}$

از آنجایی که $S^2 - 2P$ جزء گنگ ندارد (به شرط گنگ نبودن a) پس حتماً: $12\sqrt{2} = 4\sqrt{9 - a}$

$\Rightarrow \sqrt{9 - a} = 2\sqrt{2} \quad 9 - a = 8 \quad \boxed{a = 1}$

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$x^2 = t \quad t^2 - 7t - 8 = 0 \quad \Delta = 49 + 32 = 81 \quad t = \frac{7 \pm \sqrt{81}}{2}$

$\Rightarrow$ یک جواب t مثبت داره. $\Rightarrow$ دو جواب قرینه دارد. $\Rightarrow S = 0, P = -t$

$\Rightarrow 2P^2 - 3SP + 2S = 2P^2 = 2\left(\frac{7 + \sqrt{81}}{2}\right)^2 = 2\left(\frac{49 + 81 + 14\sqrt{81}}{4}\right)$

$= 65 + 7\sqrt{81}$

S_2

① $2x^2 - ax - b = 0$ $\quad S_1 = \alpha + \beta + 1 = \frac{a}{2} - \frac{1}{2} + 1 = \frac{1}{2} \Rightarrow a = 1$

② $2ax^2 + ax - a = 0$ $\quad S_2 = \alpha + \beta = -\frac{a}{2a} = -\frac{1}{2}$

$P_2 = \alpha\beta = \frac{-4}{2} = -2$ $\quad P_1 = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha + \beta) + \frac{1}{4}$ ۶

$= -2 - \frac{1}{4} + \frac{1}{4} = -2 = \frac{b}{2}$ $\quad b = -4$ $\quad \left[\frac{(-4)(1)}{4}\right] = \left[-\frac{2}{2}\right]^2 = -2$

$2\alpha(2\beta^2 + \alpha^2 - \beta) = 17$ $\quad 2\alpha(\beta^2 + \alpha^2 + \beta^2 - \beta) = 2\alpha(\underbrace{(\alpha^2 + \beta^2)}_{S^2 - 2P} + \beta(\beta - 1)) = 17$

$\xrightarrow{x = \beta}$ $a\beta^2 - a\beta - b = 0$ $\quad a\beta(\beta - 1) = b$ $\quad \beta(\beta - 1) = \frac{b}{a}$

$S = -\frac{-a}{a} = 1$ $\quad P = -\frac{b}{a}$ $\Rightarrow 2\alpha(S^2 - 2P + \beta(\beta - 1)) = 2\alpha(1 + \frac{2b}{a} + \frac{b}{a}) = 17$ ✓۷

$2\alpha(1 + \frac{2b}{a}) = 17$ $\quad 1 + \frac{2b}{a} = \frac{17}{2\alpha}$ $\quad \frac{2b}{a} = -\frac{3}{2\alpha}$ $\quad b = -\frac{a}{2\alpha}$

اختلاف دو ریشه $= \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 - 4ab}}{|a|} = \frac{\sqrt{a^2 + 4|a|(\frac{a}{2\alpha})}}{|a|} = \frac{|a|\sqrt{(1 + \frac{2}{2\alpha})}}{|a|} = \sqrt{\frac{4}{\alpha}}$

$x^2 - (a+1)x + a = 0$ $\quad x = 2\alpha - 1, x = 2\alpha + 1$ $\quad S = 4\alpha - a + 1$ $\quad P = 4\alpha^2 - 1 = a$

$4\alpha = 4\alpha^2 - 1 + 1$ $\quad 4\alpha(\alpha - 1) = 0$ $\quad \alpha = 0, 1$ $\quad \alpha = 0$ $\quad a = -1 \Rightarrow x^2 - 1 = 0$ $x = \pm 1$

$\Rightarrow -1 \notin \mathbb{N}$ ✗ $\Rightarrow \alpha = 1$ $\quad a = 3$ $\quad x^2 - 4x + 3 = (x-3)(x-1) = 0$ $\quad x = 1, 3 \in \mathbb{N}$ ✓

$\Rightarrow P = 3$ $\quad x^2 - 10x + b = 0$ $\quad x = 2\beta, 2\beta + 2$ $\quad S' = 10 = 4\beta + 2$ $\quad \beta = 2$

$\Rightarrow x = 4, 6$ $\quad P' = 24$ $\quad P' - P = 24 - 3 = 21$

$\Delta > 0$ $\quad 1 + 4m^2 + 4 > 0$ $\quad 4m^2 > -5$ $\quad m^2 > -\frac{5}{4}$ همیشه درست است

$\alpha^2 + \beta^2 = S^2 - 2P$ $\quad S = -1$ $\quad P = -m^2 - 1 \Rightarrow S^2 - 2P = 1 + 2m^2 + 2$

$= 2m^2 + 3 \xrightarrow{min} m = 0 \Rightarrow 2m^2 + 3 = 2 + 0 + 3 = 3$ ۹

$a(x - \alpha)(x - \beta) = ax^2 - \overbrace{a(S)}^{b}x + \overbrace{Pa}^{c}$ $\quad S = \alpha + \beta = -a + 2 = -2$

$\alpha^2 + \beta^2 = S^2 - 2P = 4 - 2P = 5$ $\quad 2P = -1$ $\quad P = -\frac{1}{2}$ $\quad x_{min} = -1$

$y_{min} = 1$ $\quad a - a(-1)(-c) + a(-\frac{1}{c}) = a - 2a - \frac{1}{2}a = -\frac{3}{2}a = 1$ ۱۰

$a = -\frac{2}{3}$ $\quad c = a.P = (-\frac{2}{3})(-\frac{1}{2}) = \frac{1}{3}$