

$$1x^2 + y^2 - 2x + 2y + k = 0 \rightarrow 1x^2 - 2x + y^2 + 2y + k = 0$$

$$\Rightarrow 1(x-1)^2 + (y+1)^2 + k = 0 \Rightarrow k = 11$$

$$1x^2 - 2x + 1 + y^2 + 2y + 1 + k = 0$$

$$\Rightarrow 1 + 1 + 1 = -k \Rightarrow k = 11$$

$$f(x) = \begin{cases} x^2 + 2x & ; x \geq a \\ 2x + a + 1 & ; x \leq a \end{cases} \rightarrow x^2 + 2x = 2x + a + 1 \xrightarrow{x=a} a^2 + 2a = 2a + 1$$

$$\rightarrow a^2 + a - 1 = 0 \rightarrow (a+1)(a-1) = 0 \begin{cases} a = +1 \text{ فوق} \\ a = -1 \text{ تحت} \end{cases} \Rightarrow f(x) = \begin{cases} x^2 + 2x & ; x \geq 1 \\ 2x - 1 & ; x \leq 1 \end{cases}$$

$$\Rightarrow f(1) = 1 + 2 = 3$$

$$f(x) = \begin{cases} 2x^2 - b & ; |x+1| \geq 2 \rightarrow x+1 \geq 2 \text{ یا } x+1 \leq -2 \\ a + 2x & ; -3 \leq x \leq -1 \end{cases}$$

$$x = -3 \rightarrow 18 - b = a - 4 \Rightarrow a + b = 22$$

$$y = \sqrt{4x-a} + \sqrt{b-3x} \quad D_f = \{1\}$$

یعنی در نقطه ۱ فقط تابع زیست و در بقیه نقاط زیر را می‌توان منفی است

$$\rightarrow \sqrt{12-a} + \sqrt{b-9} \Rightarrow \begin{cases} b=9 \\ a=12 \end{cases} \Rightarrow \frac{b}{a} = \frac{9}{12} = \frac{3}{4}$$

$$\text{الف) } y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+6x+9}}$$

$$\frac{x+1}{|x-3|} \geq 0 \rightarrow \frac{-1}{-3+3+} \Rightarrow D_f = [-1, 3) \cup (3, 4]$$

$$\text{ب) } y = \sqrt{[x]-4} + \sqrt{4-[x]}$$

$$[x]-4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4$$

$$4-[x] \geq 0 \rightarrow x \leq 3$$

$$D_f = \emptyset$$

الف) $y = \sqrt{\frac{x^2 - x - 2}{x^2 + 13x + 36}} \rightarrow \frac{(x-2)(x+1)}{(x^2+1)(x^2+9)} \geq 0 \rightarrow \frac{(x-2)(x+1)}{(x+1)(x-1)(x-3)(x+3)} \geq 0$
 $\frac{-2 \quad -1 \quad 1 \quad 3}{+ \quad - \quad - \quad +} \rightarrow D_f = (-\infty, -3) \cup (-1, 1) \cup (3, +\infty)$

ب) $y = \sqrt{\frac{x^3 - 2x^2 - 5x + 6}{x^3 + 11x^2 + 18x - 6}} \rightarrow \frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)} \rightarrow \frac{-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3}{+ \quad - \quad - \quad + \quad - \quad +} \rightarrow D_f = (-\infty, -3) \cup (-1, -\frac{1}{2}) \cup [\frac{1}{2}, 1) \cup [3, +\infty)$

الف) $y = \sqrt{[-x] - 1} \rightarrow [-x] - 1 \geq 0 \rightarrow [-x] \geq 1 \Rightarrow D_f = (-\infty, -1]$

ب) $y = \frac{\omega x^2 + 1}{[x]^2 - 2[x] - 3} \rightarrow \frac{\omega x^2 + 1}{([x]+1)([x]-3)} \rightarrow [x]+1 \neq 0 \quad [x] \neq -1 \Rightarrow x \notin [-1, 0)$
 $[x] - 3 \neq 0 \rightarrow x \notin [3, 4)$
 $D_f = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$

الف) $y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \frac{\sin x \neq 0 \rightarrow k\pi}{\cos x \neq 0 \rightarrow k\pi + \frac{\pi}{2}} \rightarrow D_f = \mathbb{R} - \{k\pi, k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2}\}$
 $\tan x \neq -1 \rightarrow k\pi + \frac{3\pi}{4}$

ب) $y = \sqrt{1 - \frac{1}{\sin^2 x}} \rightarrow \frac{1}{\sin^2 x} \leq 1 \rightarrow \frac{1}{\sin^2 x} \leq 1 \Rightarrow \sin^2 x \geq 1 \Rightarrow \sin x = \pm 1 \Rightarrow D_f = [2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2}]$

الف) $y = \sqrt{1 - \log_{\frac{1}{x}} x - 1} \Rightarrow \frac{x-1}{1} > 0 \Rightarrow x > 1$
 $1 - \log_{\frac{1}{x}} x - 1 \geq 0 \rightarrow \log_{\frac{1}{x}} x \leq 1 \rightarrow x-1 \leq \frac{1}{x} \Rightarrow x \leq \frac{x}{x-1} \Rightarrow D_f = (1, \frac{2}{x-1}]$

ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_{\frac{1}{x}} x} \rightarrow \frac{x^2 - x \geq 0 \rightarrow x(x-1) \geq 0}{1 - \log_{\frac{1}{x}} x \neq 0 \rightarrow \log_{\frac{1}{x}} x \neq 1 \rightarrow x-1 \neq \frac{1}{x} \rightarrow x^2 - x - 1 \neq 0}$
 $x^2 - x - 1 \neq 0 \rightarrow x \neq \frac{1 \pm \sqrt{5}}{2}$

الف) $y = \sqrt{p^{fm-n} - 1} \rightarrow p^{fm-n} \geq 1 \rightarrow p^{fm-n} \geq p^1$
 $fm - n \geq 1 \rightarrow -n^2 + fm - 3 \geq 0 \rightarrow n^2 - fm + 3 \leq 0$
 $(n-1)(n-3) \leq 0 \rightarrow \frac{1}{+ \quad - \quad - \quad +} \rightarrow D_f = [1, 3]$

ب) $y = \left(\frac{wn+d}{wn+f}\right)!$ $\frac{wn+d}{wn+f} \in \mathbb{W} \Rightarrow n \in \frac{-fW+d}{fW-f}$
 $D_f = \{n | n = \frac{-fK+d}{fW-f}, K \in \mathbb{W}\}$