

۱۹.۲۵

$$2x^2 + y^2 - 4x + 4y + k = 0 \xrightarrow{\text{دست کردن}} 2x^2 - 4x + y^2 + 4y + k = 0 \rightarrow 2(x^2 - 2x) + y^2 + 4y + k = 0$$

$$\xrightarrow{\text{تکمیل}} \begin{cases} 2(x^2 - 2x) \rightarrow 2((x-1)^2 - 1) = 2(x-1)^2 - 2 \\ y^2 + 4y \rightarrow (y+2)^2 - 4 \end{cases} \xrightarrow{\text{نکته}} 2(x-1)^2 + (y+2)^2 - 4 - 2 + k = 0$$

$$\Rightarrow 2(x-1)^2 + (y+2)^2 = 11 - k$$

برای تابع شدن معادله نقطه (۳، ۱) را درج می‌کنیم. $K = 11$ بدست می‌آید.

همواره $\Rightarrow 11 - k \geq 0 \Rightarrow K \leq 11$

۲

$$f(x) = \begin{cases} x^2 + 4x & x \geq a \\ 2x + a + 2 & x < a \end{cases} \xrightarrow{x=a} a^2 + 4a = 2a + a + 2 \Rightarrow a^2 + a - 2 = 0$$

$$a + b = ? \Rightarrow a = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm 2}{2} \Rightarrow \begin{cases} a = -2 \text{ غلط} \\ a = 1 \text{ صحیح} \end{cases} \xrightarrow{\text{نکته}} a \geq 0$$

$$\Rightarrow f(x) = \begin{cases} x^2 + 4x & x \geq 1 \\ 2x + 3 & x < 1 \end{cases} \xrightarrow{f(1)} (1)^2 + 4(1) = 5 \Rightarrow f(1) = 5$$

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$$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases} \xrightarrow{x=-3} f(x) = \begin{cases} 2(-3)^2 - b = a + 2(-3) \\ \Rightarrow 18 - b = a - 6 \end{cases}$$

$$a + b = ? \Rightarrow 18 + 4 = a + b = 22$$

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$$y = \sqrt{4x-a} + \sqrt{b-3x} \Rightarrow D_y = \begin{cases} 4x-a \geq 0 \Rightarrow x \geq \frac{a}{4} \\ b-3x \geq 0 \Rightarrow x \leq \frac{b}{3} \end{cases} \xrightarrow{\text{نکته}} D_y = \{x\} \rightarrow \frac{a}{4} \leq \frac{b}{3}$$

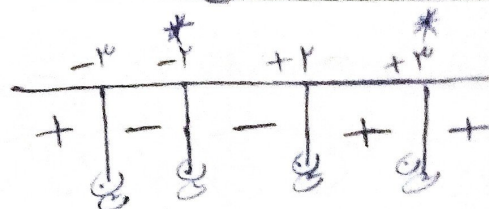
$$\frac{b}{a} = ? \Rightarrow \frac{a}{4} = \frac{b}{3} = 2 \Rightarrow \begin{cases} a = 8 \\ b = 4 \end{cases} \Rightarrow \frac{b}{a} = \frac{4}{8} = \frac{1}{2}$$

۵

الف) $y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}} \rightarrow \begin{cases} \text{I} \frac{x+1}{|x-3|} \geq 0 \xrightarrow{x \neq 3} \text{I} \cup \text{II} \\ \text{II} \frac{4-x}{x^2+2x+5} \geq 0 \xrightarrow{(x+1)^2} \text{I} \cap \text{II} \end{cases}$

ب) $y = \sqrt{x-4} + \sqrt{2-x} \rightarrow \begin{cases} \text{I} x-4 \geq 0 \rightarrow x \geq 4 \\ \text{II} 2-x \geq 0 \rightarrow x \leq 2 \end{cases} \Rightarrow D_f = \emptyset$

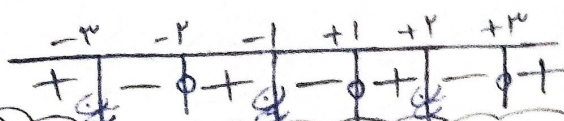
$$\text{الف) } y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 44}} = \sqrt{\frac{(x-3)(x+2)}{(x^2-9)(x^2-4)}} \xrightarrow{\text{تفكيك المقام}}$$



2

$$D_f = (-\infty, -3) \cup (-2, 2) \cup (2, 3) \cup (3, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{x^3 - 2x^2 - 2x + 4}{x^2 + 2x^2 - 2x - 4}} = \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$



$$D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup (3, +\infty)$$

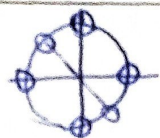
$$\text{الف) } y = \sqrt{[-x] - 2} \xrightarrow{\text{إدخال } x} [-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow [3] \geq 2 \rightarrow 3 \geq 2$$

$$\Rightarrow -x \geq 2 \rightarrow x \leq -2 \Rightarrow D_f = (-\infty, -2]$$

2

$$\text{ب) } y = \frac{2x^2 + 4}{[x]^2 - 2[x] - 3} \xrightarrow{\text{بشرط } [x]^2 - 2[x] - 3 \neq 0} [x]^2 - 2[x] - 3 \neq 0 \Rightarrow \boxed{\text{لا يوجد حله}}$$

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \frac{\frac{1}{\tan x} + 1}{\tan x + 1} \xrightarrow{\text{بشرط } \tan x \neq -1} \begin{cases} \sin x \neq 0 \\ \cos x \neq 0 \\ \tan x \neq -1 \end{cases}$$



$$D_f = \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi + \frac{\pi}{4}, \frac{5\pi}{4} \right\}$$

$$\text{ب) } y = \sqrt{1 - 4\sin^2 x} \rightarrow 1 - 4\sin^2 x \geq 0 \rightarrow 4\sin^2 x \leq 1 \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$$



$$D_f = \left[2k\pi - \frac{\pi}{4}, 2k\pi + \frac{\pi}{4} \right] \cup \left[2k\pi + \pi - \frac{\pi}{4}, 2k\pi + \pi + \frac{\pi}{4} \right]$$

$$\text{الف) } y = \sqrt{1 - \log_{\frac{1}{2}} x} \xrightarrow{\text{بشرط } 1 - \log_{\frac{1}{2}} x \geq 0} 1 - \log_{\frac{1}{2}} x \geq 0 \rightarrow \log_{\frac{1}{2}} x \leq 1 \rightarrow x^{-1} \leq \frac{1}{2} \rightarrow x \geq \frac{2}{1}$$

$$\text{II) } \log_{\frac{1}{2}} x \rightarrow x - 1 \geq 0 \rightarrow x \geq 1 \quad \text{I} \cap \text{II} \Rightarrow x \geq 2$$

$$D = \left[\frac{2}{1}, +\infty \right)$$

$$\text{ب) } y = \sqrt{\frac{x^2 - x}{1 - \log_{\frac{1}{2}} x}} \rightarrow \begin{cases} \text{I) } x^2 - x = x(x-1) \geq 0 \rightarrow x \leq 0 \text{ or } x \geq 1 \\ \text{II) } \log_{\frac{1}{2}} x \rightarrow x^2 - 3x \geq 0 \rightarrow x(x-3) \geq 0 \rightarrow x \leq 0 \text{ or } x \geq 3 \\ \text{III) } 1 - \log_{\frac{1}{2}} x \neq 0 \rightarrow \log_{\frac{1}{2}} x \neq 1 \rightarrow x^2 - 2x \neq 4 \rightarrow x^2 - 2x - 4 \neq 0 \end{cases}$$

$$\text{الف) } y = \sqrt{\frac{f(x) - x^2}{x^2 - 1}} \rightarrow \frac{f(x) - x^2}{x^2 - 1} \geq 0 \rightarrow \frac{f(x) - x^2}{x^2 - 1} \geq 1 = \frac{2}{1} \Rightarrow f(x) - x^2 \geq 2 \rightarrow -x^2 + f(x) - 2 \geq 0$$

$$\Rightarrow x = \frac{-f \pm \sqrt{f^2 - 12}}{-2} = \frac{-f \pm 2}{-2} < \frac{+3}{+1} \rightarrow \frac{+1}{-} \frac{+3}{+} \Rightarrow D_f = [1, 3]$$

$$\text{ب) } y = \left(\frac{3x+5}{2x+4} \right)! \rightarrow \left(\frac{3x+5}{2x+4} \right) \in \mathbb{W} \Rightarrow x = -\frac{f(x) - 5}{2x - 4} = \frac{f(x) - 5}{2 - 4x}$$

$$\Rightarrow D_f = \{x \mid x = k, k \in \mathbb{W}, k \neq \frac{f(x) - 5}{2 - 4x}\}$$

$$u = \frac{f(x) - 5}{2 - 4x} \quad k \in \mathbb{W}$$

بعض نواتج

1, 5

9 سوال

$$\begin{aligned}
 & \text{I} \quad x^2 - x - x(x-1) > 0 \rightarrow \frac{0}{+ \quad - \quad - \quad +} \\
 & \text{II} \quad \log_f x^{x^2-3x} \rightarrow x^2-3x > 0 \rightarrow x(x-3) > 0 \rightarrow \frac{0}{+ \quad - \quad - \quad +} \\
 & \text{III} \quad 1 - \log_f x^{x^2-3x} \neq 0 \rightarrow \log_f x^{x^2-3x} \neq 1 \rightarrow x^2-3x \neq f \rightarrow x^2-3x-f \neq 0 \\
 & \rightarrow (x-f)(x+1) \neq 0 \rightarrow \begin{cases} x \neq f \\ x \neq -1 \end{cases}
 \end{aligned}$$

$$\text{I} \cap \text{II} \cap \text{III} = D_f \Rightarrow D_f = (-\infty, -1) \cup (-1, 0) \cup (3, f) \cup (f, +\infty)$$

7 سوال

$$y = \frac{2x^2+4}{[x]^2-1[x]-3} \rightarrow \text{مقام } \oplus \text{ مخرج}$$

$$[x] = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm 3}{2} \Rightarrow \begin{cases} +2 \\ -1 \end{cases}$$

$$\begin{aligned}
 [x] = -1 & \rightarrow -1 \leq x < 0 \\
 [x] = 2 & \rightarrow 2 \leq x < 3
 \end{aligned}$$

$$D_f = \mathbb{R} - \{ \text{مخرج } \frac{0}{0} \} =$$

$$D_f = \mathbb{R} - \{ [-1, 0) \cup [2, 3) \}$$