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$a^2+4a = 2a+a+2$

$a^2+a-2=0$

$a=1 \quad a=-2$

$\left.\begin{matrix} 1\geq 1 \\ 1\geq -2 \end{matrix}\right\} \Rightarrow f(1) = x^2+4x = 5$

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$x+1 \geq 2$

$x+1 \leq -2$

$x \leq -3$

$f(-3) = 18-b = a-4$

$a+b = 24$

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$4x-a \geq 0$

$x \geq \frac{a}{4}$

$x_0 = \frac{a}{4} \quad a=12$

$b-3x \geq 0$

$x \leq \frac{b}{3}$

$x = 2 = \frac{b}{3} \quad b=6$

$\frac{b}{a} = \frac{6}{12} = \frac{1}{2}$

۵- الف)

$x+1 \geq 0$

$x \geq -1$

$x \neq 3$

$x^2+2x+4=0$

$\Delta = 4-16 < 0$

$x^2+2x+4 > 0$

$4-x \geq 0$

$x \leq 4$

$D_f = [-1, 4] - \{3\}$

ب)

$[x]-4 \geq 0$

$[x] \geq 4$

$x \geq 4$

$2-[x] \geq 0$

$[x] \leq 2$

$x < 3$

$D_f = \varnothing$

۶- الف)

$\frac{(x-3)(x+2)}{(x^2-4)(4-x^2)} = \frac{(x-3)(x+2)}{(x-2)(x+2)(3-x)(3+x)} \leq 0$


$D_f = (-\infty, -3) \cup ((2, +\infty) - \{3\})$

ب)

$x^3-2x^2-5x+6 \mid x-1$

$-x^3+x^2$

$-x^2-5x+6$

$+x^2-x$

$-6x+6$

$+6x-6$

$0$

$x^2-x-6$

$x^3+2x^2-5x-6 \mid x+1$

$-x^3-x^2$

$x^2-5x-6$

$-x^2-x$

$-6x-6$

$+6x+6$

$0$

$x^2+x-6$

$\frac{(x-1)(x-3)(x+2)}{(x-2)(x+3)(1+x)} \leq 0$


$D_f = (-\infty, -3) \cup [-2, -1) \cup (1, 2] \cup [3, +\infty)$

۷-

الف) $[-x]-2 \geq 0$

$[-x] \geq 2$

$-x \geq 2 \quad x \leq -2$

$D_f = (-\infty, -2]$

ب) $[x]^2 - 2[x] - 3 = 0$

$([x]-3)([x]+1)=0$

$[x]=3 \quad [x]=-1$

$3 \leq x < 4 \quad -1 \leq x < 0$

$D_f = \mathbb{R} - \{[3,4) \cup [-1,0)\}$

۸-

الف) $x \neq \frac{k\pi}{2}$

$\tan x + 1 = 0$

$x \neq k\pi + \frac{3\pi}{4}$

$D_f = \mathbb{R} - \left\{ \frac{k\pi}{2} \cup \left\{ k\pi + \frac{3\pi}{4} \right\} \right\}$

ب) $1 - 4\sin^2 x \geq 0$

$4\sin^2 x \leq 1$

$\sin^2 x \leq \frac{1}{4}$

$-\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$D_f = [k\pi + \frac{5\pi}{6}, k\pi + \frac{7\pi}{6}]$

۹-

الف) $1 - \log_{\frac{1}{2}}^{x-1} \geq 0$

$\log_{\frac{1}{2}}^{x-1} \leq 1$

$x-1 > 0$

$x > 1$

$x-1 \leq \frac{1}{2}$

$x \leq \frac{3}{2}$

$D_f = (1, \frac{3}{2}]$

ب) $x(x-1) \geq 0$

| | 0 | 1 | |
|---|---|---|---|
| | + | − | + |

$1 - \log_4^{x^2-3x} = 0$

$\log_4^{x^2-3x} = 1$

$x^2 - 3x = 4 \quad x^2 - 3x - 4 = 0$

$x = -1 \quad x = 4$

$x(x-3) > 0$

| | 0 | 3 | |
|---|---|---|---|
| | + | − | + |

$D_f = \left((-\infty, 0) - \{-1\}\right) \cup \left((3, +\infty) - \{4\}\right)$

۱۰-

الف) $4^{4x-x^2} \geq 8$

$4x - x^2 \geq 3$

$x^2 - 4x + 3 \leq 0$

| | 1 | 3 | |
|---|---|---|---|
| | + | − | + |

$D_f = [1, 3]$

ب) $\frac{3x+5}{2x+4} \in \mathbb{Z}$

$x \in -\frac{4\mathbb{Z}-5}{2\mathbb{Z}-3}$

$x \in \frac{5-4\mathbb{Z}}{2\mathbb{Z}-3}$