

$$2x^2 + y^2 - 4x + 4y + k = 0 \Rightarrow \begin{cases} 2x^2 - 4x + 0 \rightarrow \text{مربع کامل} \Rightarrow 2(x-2x+1) = 0 = 2 \\ 0^2 + 4y + 4 \rightarrow \text{مربع کامل} \Rightarrow (y+2)^2 = 4 \Rightarrow \Delta = 4 \end{cases}$$

$$\Rightarrow k = 0 + 4 \Rightarrow \underline{k = 4}$$

$$f(x) = \begin{cases} x^2 + 4x; & x \geq a \\ 2x + a + 2; & x < a \end{cases} \quad \begin{aligned} & \text{a) } a^2 + 4a = 2(a+a+2) \Rightarrow a^2 + a - 2 = 0 \\ & \Rightarrow (a-1)(a+2) = 0 \Rightarrow a = 1 \text{ (✓) } \text{ یا } a = -2 \text{ (✗) } \end{aligned}$$

$$\Rightarrow f(x) = \begin{cases} x^2 + 4x; & x \geq 1 \\ 2x + 2; & x < 1 \end{cases} \quad \text{b) } f(1) = 5$$

$$y = \sqrt{4x-a} + \sqrt{b-2x} \xrightarrow{\text{D.S.}} \sqrt{12-a} + \sqrt{b-6} \Rightarrow \begin{cases} a \geq 12 \\ b \geq 6 \end{cases} \Rightarrow \frac{b}{a} \geq \frac{1}{2}$$

$$f(x) = \begin{cases} 2x^2 - b; & |x+1| \geq 2 \Rightarrow x+1 \geq 2 \text{ و } x+1 \leq -2 \Rightarrow x \geq 1 \text{ و } x \leq -3 \\ a+2x; & -3 < x < -1 \end{cases}$$

$$x = -3 \Rightarrow 11 - b = a + (-6) \Rightarrow \underline{b+a = 17}$$

الف) $\sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{5-x}{x^2+2x+4}}$ $\text{D.S.} = \begin{cases} \frac{-1}{-3+3} + \frac{5}{(-\infty, 4]} \Rightarrow \text{D.S.} = \emptyset \end{cases}$

ب) $\sqrt{[x]-4} + \sqrt{2-[x]}$ $\Rightarrow \begin{cases} [x] \geq 4 \Rightarrow x \geq 4 \\ [x] \leq 2 \Rightarrow x < 3 \end{cases} \Rightarrow \text{D.S.} = \emptyset$

الف) $\sqrt{\frac{x^2 - x - 6}{x^2 - 12x^2 + 24}}$ صورت $\begin{cases} (x-2)(x-3) < 0 \\ +2-12+24 > 0 \end{cases} \Rightarrow (x-2)(x-3) < 0 \Rightarrow \begin{matrix} t=2 \rightarrow x=2-2 \\ t=3 \rightarrow x=3-2 \end{matrix}$

ب) $\frac{-2-2}{+2-2+2+2} \Rightarrow Df = (-\infty, -3) \cup (-2, 2) \cup (2, 3) \cup (3, +\infty)$

ج) $\sqrt{\frac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}} = \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x-2)(x+3)}} \Rightarrow \frac{-2-2-1}{+2-2+2+2} \Rightarrow Df = (-\infty, -3) \cup [-2, -1) \cup [2, 3) \cup (3, +\infty)$

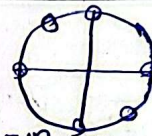
الف) $\sqrt{[-x]-2} \Rightarrow [-x]-2 \geq 0 \Rightarrow [-x] \geq 2 \Rightarrow -x \geq 2 \Rightarrow x \leq -2$

ب) $Df = (-\infty, -2]$

ج) $[x] = t \Rightarrow t^2 - 2t - 3 = (t-3)(t+1) \Rightarrow \begin{cases} t=3 \rightarrow [x] = 3 \Rightarrow x \in [3, 4) \\ t=-1 \rightarrow [x] = -1 \Rightarrow x \in [-1, 0) \end{cases}$

د) $\frac{-1-0}{+2-2+2+2} \Rightarrow Df = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$

الف) $\frac{\cot x + 1}{\tan x + 1} \Rightarrow \begin{cases} \sin x, \cos x \neq 0 \\ \tan x + 1 \neq 0 \Rightarrow \tan x \neq -1 \end{cases} \Rightarrow$



$\Rightarrow Df = \mathbb{R} - \{k\pi, k\pi - \frac{\pi}{4}\}$

ب) $\sqrt{1 - \sin^2 x} \Rightarrow 1 - \sin^2 x \geq 0 \Rightarrow \sin^2 x \leq 1 \Rightarrow \sin^2 x \leq \frac{1}{4} \Rightarrow \frac{1}{2} |\sin x| \leq \frac{1}{4}$

د) $\Rightarrow Df = [k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}]$

الف) $\sqrt{1 - \log_{\frac{1}{f}} x - 1} \Rightarrow \begin{cases} x - 1 \geq 0 \Rightarrow x \geq 1 \\ 1 - \log_{\frac{1}{f}} x - 1 \geq 0 \Rightarrow \log_{\frac{1}{f}} x \leq 0 \Rightarrow x - 1 \geq 1 \Rightarrow x \geq 2 \end{cases}$

ب) $\frac{\sqrt{x(x-1)}}{1 - \log_{\frac{1}{f}} x - 1} \Rightarrow \begin{cases} x(x-1) \geq 0 \Rightarrow x \geq 1 \text{ or } x \leq 0 \text{ (1)} \\ x^2 - x \geq 0 \Rightarrow x(x-1) \geq 0 \Rightarrow x \geq 1 \text{ or } x \leq 0 \text{ (2)} \\ \log_{\frac{1}{f}} x \neq 1 \Rightarrow x \neq f \Rightarrow x^2 - x - f = (x-f)(x+1) \Rightarrow x \neq f-1 \text{ (3)} \end{cases}$

$\Rightarrow Df = \mathbb{R} - [0, 3] - \{f-1\}$

الف) $\sqrt{\frac{x^2 - x^2}{x^2 - x^2} - 1} \Rightarrow \begin{cases} x^2 - x^2 \geq 0 \Rightarrow -x(x-1) \geq 0 \Rightarrow \frac{0}{x^2 - x^2} \text{ (1)} \\ x^2 - x^2 - 1 \geq 0 \Rightarrow x^2 - x^2 \geq 1 \Rightarrow \log_{\frac{1}{f}} x \leq x^2 - x^2 \text{ (2)} \end{cases} \Rightarrow Df = [1, 3]$

ب) $(\frac{3x+d}{x+d}) \in W$

$\Rightarrow 3x+d \in W = 3x+d \Rightarrow 3x+d - 3x = -3x+d$

$\Rightarrow x(3x-3) = -3x+d \Rightarrow x = \frac{-3x+d}{3x-3}$

$\Rightarrow -x^2 + 3x \geq 3 \Rightarrow -x^2 + 3x - 3 \geq 0 \Rightarrow -(x-2)(x-1) \geq 0$

$\Rightarrow \frac{1}{x^2 - 3x - 3}$

$\Rightarrow Df = \{x | x = \frac{-3k+d}{3k-3}, k \in W\}$