

۱۸, ۲۵

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$$\begin{aligned}
 &rx^2 + y^2 - 4x + 6y + k = 0 \rightarrow rx^2 - 4x + y^2 + 6y + k = 0 \rightarrow r(x^2 - 4x) + (y^2 + 6y) + k = 0 \\
 &\rightarrow r((x-1)^2 - 1) + (y+3)^2 - 9 + k = 0 \rightarrow r(x-1)^2 - r + (y+3)^2 - 9 + k = 0 \rightarrow \dots \rightarrow 15 \\
 &\rightarrow r(x-1)^2 + (y+3)^2 - 11 + k = 0 \rightarrow r(x-1)^2 + (y+3)^2 = 11 - k \rightarrow 11 - k \geq 0 \\
 &\rightarrow k \leq 11
 \end{aligned}$$

تابع فقط (۳-۱) فقط به ازای $k = 11$ قابل قبول است

$$f(x) = \begin{cases} x^2 + rx \rightarrow x \geq a \\ rx + a + r \rightarrow x \leq a \end{cases} \rightarrow \begin{aligned} &a^2 + ra = ra + a + r \\ &a^2 + ra = ra + r \\ &a^2 + a - r = 0 \\ &(a-1)(a+r) = 0 \rightarrow a \begin{cases} -r \rightarrow \text{غیر} \rightarrow a > 0 \\ 1 \rightarrow \checkmark \end{cases} \end{aligned}$$

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$$f(1) = f(a) = a^2 + ra = ra + a + r = 8 \quad \checkmark$$

$$f(x) = \begin{cases} rx^2 - b & |x+1| \geq r \\ a + rx & -r \leq x \leq -1 \end{cases} \xrightarrow{x=-r} \begin{aligned} &rx^2 - b = a + rx \\ &1 - b = a + (-r) \\ &a + b = r \end{aligned}$$

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$$y = \sqrt{4x-a} + \sqrt{b-3x} \rightarrow \begin{cases} 4x-a \geq 0 \rightarrow x \geq \frac{a}{4} \\ b-3x \geq 0 \rightarrow x \leq \frac{b}{3} \end{cases} \xrightarrow{\text{میشود}} \begin{aligned} &\frac{a}{4} = \frac{b}{3} = r \rightarrow a = 4r \\ &\frac{b}{a} = \frac{c}{1r} = \frac{1}{r} \end{aligned}$$

۲ $D = \{r\}$

$$y = \sqrt{\frac{x+1}{|x-r|}} + \sqrt{\frac{4-x}{x^2+rx+r}} \quad \begin{aligned} &x \neq r \\ &x+1 \geq 0 \rightarrow x \geq -1 \\ &4-x \geq 0 \rightarrow x \leq 4 \\ &x^2+rx+r > 0 \rightarrow \text{همیشه مثبت} \end{aligned}$$

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$A \cap B = \{r\}$

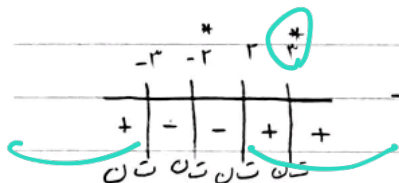
$$y = \sqrt{[x]-\varepsilon} + \sqrt{r-[x]} \rightarrow \begin{aligned} &[x]-\varepsilon \geq 0 \Rightarrow [x] \geq r \rightarrow [r, +\infty) = A \\ &r-[x] \geq 0 \Rightarrow [x] \leq r \rightarrow (-\infty, r] = B \end{aligned} \rightarrow A \cap B = \emptyset$$

Parsian

$$y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 44}} \quad x^2 - x - 4 \geq 0 \rightarrow (x-2)(x+2) \geq 0$$

(1, V)

$$x^2 - 13x^2 + 44 \geq 0 \rightarrow (x^2 - 4)(x^2 - 11) \geq 0 \rightarrow (x-2)(x+2)(x-\sqrt{11})(x+\sqrt{11}) \geq 0$$

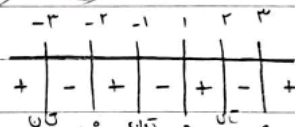


$$\rightarrow Df = (-\infty, 2) \cup (2, +\infty)$$

$$(-\infty, -2) \cup (2, +\infty) - \{2\}$$

$$y = \sqrt{\frac{x^2 - 2x^2 - 2x + 4}{x^2 + 2x^2 - 2x - 4}} \rightarrow (x-1)(x^2 - x - 4) \rightarrow (x-1)(x-2)(x+2)$$

$$\rightarrow (x+1)(x^2 + x - 4) \rightarrow (x+1)(x+2)(x-2)$$



$$Df = (-\infty, -2) \cup [-2, -1) \cup [1, 2) \cup [2, +\infty)$$

$$y = \sqrt{[-x] - 2} \rightarrow [-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2$$

(2)

$$y = \frac{[x]^2 + 4}{[x]^2 - 2[x] - 2} \rightarrow ([x] - 2)([x] + 1) \neq 0 \rightarrow \begin{cases} [x] = -1 \rightarrow [-1, 0) \cap A \\ [x] = 2 \rightarrow [2, 3) \cap B \end{cases}$$

$$Df = \mathbb{R} - \{[-1, 0) \cup [2, 3)\}$$

$$y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \tan x \neq -1 \rightarrow \frac{\sin x}{\cos x} \neq -1 \rightarrow \cos x \neq 0 \rightarrow \mathbb{R} - \left\{k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2}\right\}$$

$$\sin x \neq 0$$

(1, V)

$$y = \sqrt{1 - 5\sin^2 x} \rightarrow 1 - 5\sin^2 x \geq 0 \rightarrow 5\sin^2 x \leq 1 \rightarrow \sin^2 x \leq \frac{1}{5}$$

$$Df = \left[k\pi - \frac{\pi}{\sqrt{5}}, k\pi + \frac{\pi}{\sqrt{5}}\right] \quad -\frac{1}{\sqrt{5}} \leq \sin x \leq \frac{1}{\sqrt{5}}$$

۱) مبدا ۰ و ۱ به عنوان عوف

$$y = \sqrt{1 - \log \frac{x-1}{r}} \rightarrow \log \frac{x-1}{r} \leq 1 \rightarrow x-1 \geq \frac{1}{r} \rightarrow x \geq \frac{r}{r-1} \quad A$$

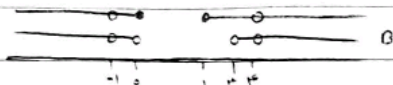
$$x-1 > 0 \rightarrow x > 1 \quad B$$

$$A \cap B = \left(\frac{r}{r-1}, +\infty \right)$$

$$y = \frac{\sqrt{x^r - x}}{1 - \log \frac{x^r - rx}{r}} \rightarrow 1 - \log \frac{x^r - rx}{r} \neq 0 \rightarrow x^r - rx \neq r \rightarrow x^r - rx - r \neq 0$$

$$x^r - rx > 0 \rightarrow x(x-r) > 0 \quad (-\infty, 0) \cup (r, +\infty) \quad B$$

$$x^r - x \geq 0 \rightarrow x(x-1) \geq 0 \quad (-\infty, 0] \cup [1, +\infty) \quad A$$

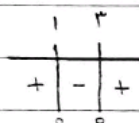


$$Df = \left((r, +\infty) \cup (-\infty, 0) \right) - \{-1, r\}$$

$$(-\infty, -1) \cup (-1, 0) \cup (r, r) \cup (r, +\infty)$$

$$y = \sqrt{r^{rx-x^r} - 1} \rightarrow r^{rx-x^r} - 1 \geq 0 \rightarrow r^{rx-x^r} \geq 1 \rightarrow rx - x^r \geq r$$

$$Df = [1, r]$$



$$rx - x^r - r \geq 0$$

$$x^r - rx + r \leq 0$$

$$(x-1)(x-r) \leq 0$$

$$y = \left(\frac{rx + a}{rx + r} \right)!$$

$$\frac{rx + a}{rx + r} \in \mathbb{N} \rightarrow rx + a = k(rx + r) \rightarrow rx + a = krx + kr$$

$$rx + a = krx + kr$$

$$Df = \left\{ x = \frac{-rk + a}{rk - r} \mid k \in \mathbb{N} \right\}$$

$$x = \frac{-rk + a}{rk - r}$$