آزمین ایزدی تکلیف ۲۰ ۱۷ یازدهم پسر A

۱) (۲)

$\frac{1}{x-1} = |x-1| \xrightarrow{x>1} \frac{1}{x-1} = x-1 \rightarrow 1 = (x-1)^2 \rightarrow x=2$ یا $x=0 \xrightarrow{x>1}$ **$x=2$**

$\xrightarrow{x<1} 1 = -(x-1)^2 \rightarrow$ غیرممکن زیرا سمت راست نمی‌تواند منفی باشد

۲) (۲)

الف) $\left|\frac{1-x^2}{x}\right| < 2 \rightarrow -2 < \frac{1-x^2}{x} < 2 \Rightarrow \frac{1-x^2}{x} > -2 \rightarrow \frac{1-x^2}{x} > 0 \rightarrow$ دینومی $x=0$ $x=\frac{1}{2}$

$\Rightarrow (-\infty, 0) \cup (\frac{1}{2}, \infty)$ ① ، $\frac{1-x^2}{x} < 2 \rightarrow \frac{-1}{x} < 0 \rightarrow (0, +\infty)$ ② $\Rightarrow$ ① ∩ ② $\Rightarrow$ $(\frac{1}{2}, +\infty)$

ب) $\left|\frac{x-2}{x^2+1}\right| > 1 \rightarrow \frac{(x-2)^2}{(x^2+1)^2} > 1 \rightarrow (x-2)^2 > (x^2+1)^2 \rightarrow x^2-4x+4 < x^4+2x^2+1$

$-x^4-4x+3 < 0 \rightarrow x^4+4x-3 > 0 \rightarrow x = \frac{-4 \pm \sqrt{16+12}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} \Rightarrow x = -2-\sqrt{7}, x = -2+\sqrt{7}$

$\rightarrow (-2-\sqrt{7}, -2+\sqrt{7}) \xrightarrow{x \neq -\frac{1}{2}} (-2-\sqrt{7}, -\frac{1}{2}) \cup (-\frac{1}{2}, -2+\sqrt{7})$ ✓

۳) (۲)

$x^2 < |x+6| \begin{cases} x \geq -6 \rightarrow x^2 < x+6 \rightarrow x^2-x-6 < 0 \rightarrow (x-3)(x+2) < 0 \rightarrow -2 < x < 3 \\ x < -6 \rightarrow x^2 < -(x+6) \rightarrow x^2+x+6 < 0 \rightarrow \Delta < 0 \end{cases}$

$|x-a| < b \Rightarrow$ $a = \frac{-2+3}{2} = \frac{1}{2}$ ، $b = \frac{3-(-2)}{2} = \frac{5}{2}$

۴)

$y = |x+1| - |2x| + |x-2| \rightarrow x \leq -1 \dots y = (-(x+1)) - (-2x) + (-(x-2))$

$\rightarrow y = 1$

$-1 < x \leq 0 \rightarrow y = x+1+2x+x-2 \rightarrow y = 4x-1$ / $x=-1 \rightarrow y=1$

$x=0 \rightarrow y=-1$

۴) ②

$0 \le x < 2 \quad y = x+1-2x-x+1 \to y = -2x+3 <$ $x=0 \to y=3$; $x=2 \to y=-1$

$x \ge 2 \to y = x+1-2x+x-2 \to y=-1 \Rightarrow$ بالاترین $\to 3$ ، کمترین $\to -1$ $\Rightarrow 3+(-1)=2$

۵) ②

$y=\frac{1}{2}|x|-2 \longrightarrow y=\frac{1}{2}|x+4|-1 \Rightarrow \frac{1}{2}|x|-2=\frac{1}{2}|x+4|-1 \xrightarrow{\times 2} |x|-4=|x+4|-2$

$\longrightarrow |x|=|x+4|+2$
- $x \ge 0 \longrightarrow x = x+6$ غیر ممکن
- $-4 \le x < 0 \longrightarrow -x = x+6 \longrightarrow x=-3$
- $x < -4 \longrightarrow -x = -x-2$ غیر ممکن

۶) ②

الف)

$f(x)=[\,|1-x|-|x+4|\,]$
- $x \le -4 \to 1-x+x+4=5$
- $-4<x<1 \to 1-x-x-4=-2x-3$
- $x \ge 1 \to x-1-x-4=-5$

$-5 \le |1-x|-|x+4| \le 5 \longrightarrow [-5,5] \Rightarrow \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

ب) $\frac{x+1}{x}=1+\frac{1}{x}$, $y=\frac{1}{x}-[1+\frac{1}{x}]$

$(t-[t]) \leftarrow \frac{1}{x}-(1+[\frac{1}{x}])$

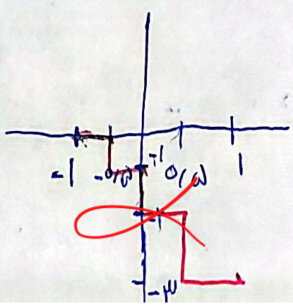
$[0,1)$ در بازه $\longrightarrow 0 \le t-[t] < 1 \to -1 \le t-[t]-1 < 0$

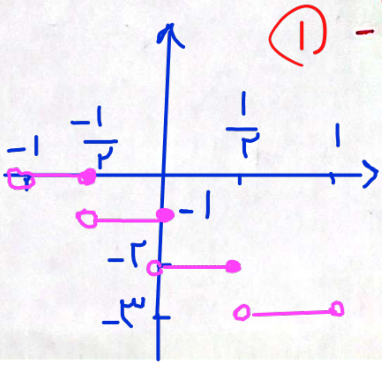
$\Rightarrow R=[-1,0)$

۷- ①

الف) $y=[-2x]-1 \quad x\in(-1,1)$

$y=\begin{cases} 0 & -1<x<-\frac{1}{2} \\ -1 & -\frac{1}{2}\le x<0 \\ -2 & 0\le x<\frac{1}{2} \\ -3 & \frac{1}{2}\le x<1 \end{cases}$

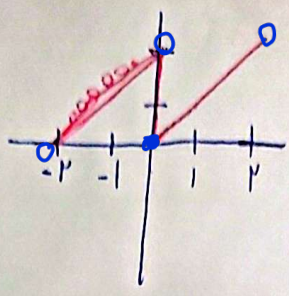




۷) آدین ایزدی

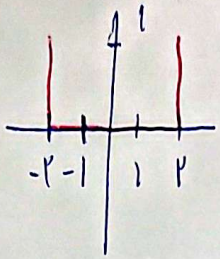
ب) $x - 2\left[\frac{x}{2}\right]$, $x \in (-2, 2)$

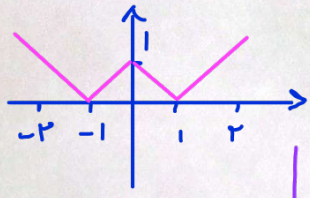
$$y = \begin{cases} x+2 & -2 < x < 0 \to -1 \\ x & 0 \le x < 2 \to 0 \end{cases}$$



۸) ۰

الف) $[||x|-1|]$, $x \in [-2, 2]$





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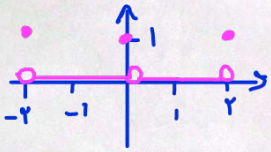
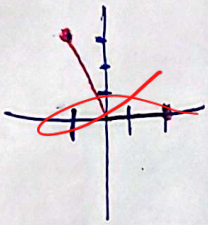
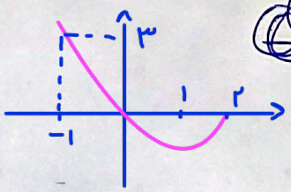


ب) $[x^2 - 2x]$, $x \in [-1, 2]$





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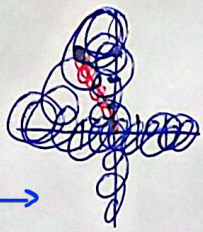

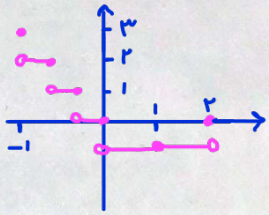

۹) ۲

$a + [b] = 1,2$, $b - a[a] = 2,4$

$a+b = ?$ $\quad [a]=m$, $[b]=n$ $\quad a = 1,2 - n$, $b = 2,4 + m$ $\quad m \le 1,2 - n < m+1$

$b = 2,4 + m$, $m+n = 1$ ← $m+n \le 2,4 < m+n+1$

$n \le 1,4 + m < n+1 \to n - m \le 2,4 < n - m + 1 \to n - m = 2$ $\quad \begin{cases} n+m=1 \\ n-m=2 \end{cases} \to n=2, m=-1$

$\to a = 1,2 - 2 = -0,8$, $b = 2,4 - 1 = 1,4$ $\quad a+b = -0,8 + 1,4 = 0,6$ ✓

۱۰) ۲

$(1+\sqrt{2})^4 + (1-\sqrt{2})^4 = 198$

$1 - \sqrt{2} < 0 \to 0 < (1-\sqrt{2})^4 < 1$ $\quad [(1+\sqrt{2})^4] + [(1-\sqrt{2})^4] = [(1+\sqrt{2})^4] + 0$

$[(1+\sqrt{2})^4] = 198 - 1 \to [(1+\sqrt{2})^4] = 197$ ✓

چون $(1-\sqrt{2})^4 > 0$