

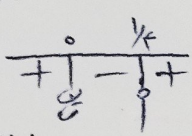
$$\frac{1}{x-1} = |x-1| \rightarrow \begin{cases} \text{قد مثبت} \oplus \rightarrow 1 = (x-1)^2 \rightarrow x^2 - 2x + 1 = 1 \rightarrow x(x-2) = 0 \rightarrow \begin{cases} x=0 \\ x=2 \end{cases} \\ \text{قد منفی} \ominus \rightarrow 1 = -(x-1)(1-x) = x - x^2 - 1 + x = -x^2 + 2x - 1 = 1 \end{cases}$$

$$\Rightarrow -x^2 + 2x - 2 = 0 \rightarrow x = \frac{-2 \pm \sqrt{4-8}}{-2} \rightarrow \Delta < 0 \rightarrow \text{فاصله ریشه حقیقی نیست}$$

$$\Rightarrow \text{تست} \rightarrow \begin{cases} x=0 \rightarrow x \text{ غلط} \\ x=2 \rightarrow x \text{ غلط} \end{cases} \rightarrow \frac{1}{0-1} = |0-1| \rightarrow -1 \neq 1 \quad (\times)$$

$x=2$ تنها ریشه صحیح است ✓

الف) $\left| \frac{x-1}{x} \right| < 2 \rightarrow -2 < \frac{x-1}{x} < 2 \rightarrow \begin{cases} \frac{x-1}{x} < 2 \rightarrow \frac{x-1-2x}{x} < 0 \rightarrow \frac{-x-1}{x} < 0 \rightarrow \frac{-1}{x} < 0 \rightarrow x > 0 \\ \frac{x-1}{x} > -2 \rightarrow \frac{x-1+2x}{x} > 0 \rightarrow \frac{3x-1}{x} > 0 \rightarrow \frac{x-1}{3} > 0 \rightarrow x > \frac{1}{3} \end{cases}$

$\Rightarrow$  $\rightarrow (-\infty, 0) \cup (\frac{1}{3}, +\infty)$

ب) $\left| \frac{x-2}{x+1} \right| > 1 \rightarrow \begin{cases} \frac{x-2}{x+1} > 1 \rightarrow \frac{x-2-x-1}{x+1} > 0 \rightarrow \frac{-x-3}{x+1} > 0 \rightarrow \frac{-x-3}{x+1} > 0 \rightarrow \frac{-3}{x+1} > 0 \rightarrow x < -1 \\ \frac{x-2}{x+1} < -1 \rightarrow \frac{x-2+x+1}{x+1} < 0 \rightarrow \frac{2x-1}{x+1} < 0 \rightarrow \frac{x-1/2}{x+1} < 0 \rightarrow x > -1 \end{cases}$

$\Rightarrow (-1, \frac{1}{2}) \cup (\frac{1}{2}, +\infty)$

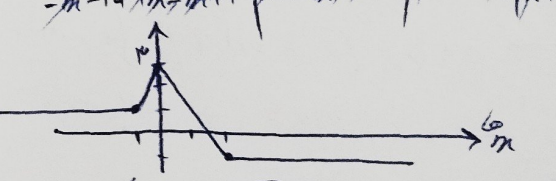
$x^2 < |x+4| \rightarrow \begin{cases} x+4 > x^2 \rightarrow x^2 - x - 4 < 0 \rightarrow (x-3)(x+2) < 0 \\ x+4 < -x^2 \rightarrow x^2 + x + 4 < 0 \rightarrow \Delta < 0 \rightarrow \text{فاصله ریشه حقیقی نیست} \end{cases}$

$|x-a| < \beta$

$a = ?$

$\rightarrow -\beta < x-a < \beta \rightarrow \begin{cases} x < a+\beta \\ x > a-\beta \end{cases} \rightarrow \begin{cases} a+\beta = 3 \\ a-\beta = -2 \end{cases} \rightarrow \frac{2a}{2} = 1 \rightarrow a = \frac{1}{2}, \beta = \frac{5}{2}$

$y = |x+1| - |2x| + |x-2| \rightarrow \begin{cases} x = -1 \\ x = 0 \\ x = 2 \end{cases}$

$\Rightarrow$ 

$y_{\max} + y_{\min} = 3 - 1 = 2$

حد اکثر چهار تابع است $y = 3$ و کمترین مقدار آن $y = -1$ است

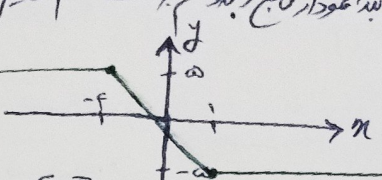
$y = \left| \frac{1}{4}x \right| - 2 \rightarrow \begin{cases} \text{خطی (مستقیم)} \\ \text{شکل (V)} \end{cases} \rightarrow y = \left| \frac{1}{4}(x+4) \right| - 2 + 1 \Rightarrow y = \left| \frac{1}{4}x + 1 \right| - 1$

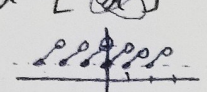
تحل تقاطع نمودار صریح و اولیه

$\begin{cases} y = \left| \frac{1}{4}x \right| - 2 \\ y = \left| \frac{1}{4}x + 1 \right| - 1 \end{cases} \rightarrow \begin{cases} \frac{1}{4}x - 2 = \frac{1}{4}x + 1 \rightarrow -2 \neq 1 \quad (\times) \\ -\frac{1}{4}x - 2 = \frac{1}{4}x + 1 \rightarrow x = -3 \quad (\checkmark) \rightarrow y = -\frac{1}{4} \\ \frac{1}{4}x - 2 = -\frac{1}{4}x - 1 \rightarrow x = 1 \rightarrow \text{غلط} \quad (\times) \end{cases}$

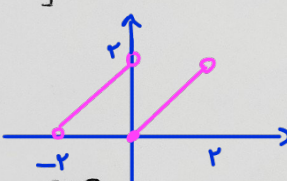
* این دو تابع در نقطه $(-3, -\frac{1}{4})$ تقاطع میکنند ✓

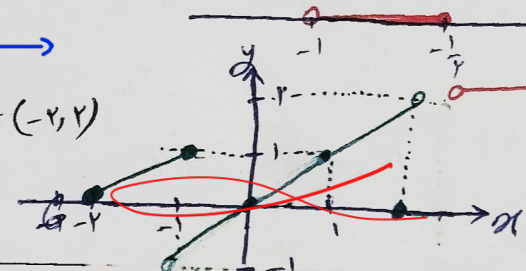
$f(x) = [1-x] - [x+4]$ * ابتدا نمودار تابع را بکشید

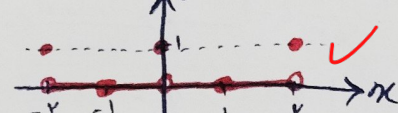
$\Rightarrow R_f = [-5, +5] \rightarrow [R_f] = \{\pm 5, \pm 4, \pm 3, \pm 2, \pm 1, 0\}$ 

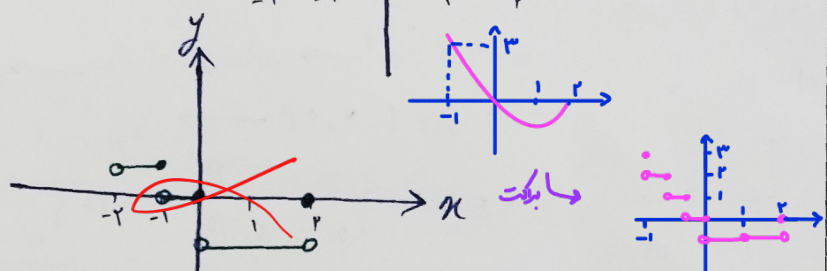
ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] \rightarrow \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] - 1 \rightarrow 0 - [0] - 1 = -1$ 

$\rightarrow R_f = [-1, 0) \quad R = [-1, 0)$ اشغال یک واحد به بالا

الف) $y = [-2x] - 1 \quad x \in (-1, 1)$ 

ب) $y = x - 2\left[\frac{x}{2}\right] \quad x \in (-2, 2)$ 

الف) $y = [1-|x|] \quad x \in [-2, 2]$ 

ب) $y = [x^2 - 2x] \quad x \in [-1, 2]$ 

$a + [b] = 4, 2 \rightarrow a + [b] = 4, 2 \xrightarrow{\mathbb{Z}} [b] \in \mathbb{Z} \rightarrow a = 3, 1$

$b - [a] = 2, 4 \rightarrow [a] \in \mathbb{Z} \rightarrow b = 3, 4$

$a + b = 8 \rightarrow a + b = [a] + 0, 2 + [b] + 0, 4 = [a] + [b] + 0, 4 = 8$

$\begin{cases} [a] + [b] = 4 \\ [b] - [a] = 2 \end{cases} \rightarrow \begin{cases} 2[b] = 4 \rightarrow [b] = 2 \\ [a] = 1 \end{cases} \rightarrow 1 + 2 + 0, 4 = (3, 2) \checkmark$

$(1+\sqrt{2})^4 + (1-\sqrt{2})^4 = 198 \rightarrow \frac{((1+\sqrt{2})^2)^2}{1+2+2\sqrt{2}} + \frac{((1-\sqrt{2})^2)^2}{1+2-2\sqrt{2}} = (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2$

$[(1+\sqrt{2})^4] = 8$ $\Rightarrow (3+2\sqrt{2})(9+1-4\sqrt{2}) + (3-2\sqrt{2})(9+1+4\sqrt{2})$

$\Rightarrow 18 - 18\sqrt{2} + 18\sqrt{2} - 24 + 18 + 18\sqrt{2} - 18\sqrt{2} - 24 \Rightarrow 2(18) - 2(24) = 2(18-24) = -12$

$\begin{cases} \sqrt{2} \approx 1,41 \\ 1 \end{cases} \rightarrow 1+\sqrt{2} = (2,41)^4 = 196,99 \rightarrow [196,99] = 196 \checkmark$