رادین ملکیان یازدهم

16,25

۱- 1,75

$$\frac{1}{x-1}=|x-1| \begin{cases} x>1: & \frac{1}{x-1}=x-1 \xrightarrow{x\neq 1} (x-1)^2=1 \Rightarrow x=\cancel{0},2 \\ x<1: & \frac{1}{x-1}=-x+1 \xrightarrow{x\neq 1} -(x-1)^2=1 \Rightarrow x^2-2x+2=0 \end{cases}$$

دقت!

$\Delta<0$

1 جواب دارد

۲- 1,5

الف) $\left|\frac{2x-1}{x}\right|<2 \Rightarrow \frac{|2x-1|}{|x|}<2 \xrightarrow{|x|>0} |2x-1|<2|x|$

$\Rightarrow (|2x-1|)^2<(2|x|)^2 \Rightarrow 4x^2-4x+1<4x^2 \Rightarrow 4x>1 \Rightarrow x>\frac{1}{4}$

ب) $\left|\frac{x-2}{2x+1}\right|>1 \Rightarrow \frac{|x-2|}{|2x+1|}>1 \Rightarrow |x-2|>|2x+1| \Rightarrow (|x-2|)^2>(|2x+1|)^2$

$\Rightarrow x^2-4x+4>4x^2+4x+1 \Rightarrow x=-\frac{9}{3},\frac{1}{3} \Rightarrow D_f=(\cancel{-\infty,-3})\cup(\frac{1}{3},+\infty)$

سوال ۳ 2

$x^2<|x+6| \xrightarrow{x>-6} x^2<x+6 \Rightarrow x^2-x-6<0 \Rightarrow (x-3)(x+2)<0$

	-2		3	
$+$		$-$		$+$

$\Rightarrow x\in(-2,3)$

$|x-\alpha|<\beta \Rightarrow -\beta<x-\alpha<\beta$

$\alpha-\beta<x<\alpha+\beta \begin{cases} \alpha+\beta=3 \\ \alpha-\beta=-2 \end{cases} \Rightarrow 2\alpha=1 \Rightarrow \alpha=\frac{1}{2} \Rightarrow \beta=\frac{5}{2}$

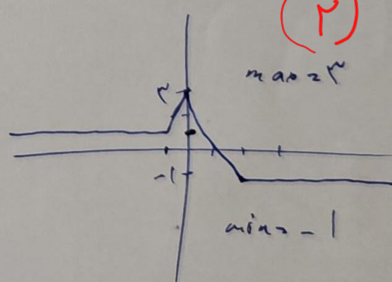
۴- 2

$x\geq 2;\ (x+1)-(2x)+(x-2)=-1$

$0<x<2;\ (x+1)-(2x)-(x-2)=-2x+3$

$-1<x\leq 0\quad (x+1)+(2x)-(x-2)=2x+3$

$x\leq -1,\quad -(x+1)+(2x)-(x-2)=+1$



max = 3

min = -1

جمع $=3+(-1)=2$

۵. $y=|\frac{1}{2}x|-1 \xrightarrow[\text{x های منفی}]{\text{۴ واحد به طرف}} |\frac{1}{2}(x+4)|-2 \xrightarrow[\text{y های مثبت}]{\text{یک واحد}} y=|\frac{1}{2}(x+4)|-2+1$

۲

$y=y'$, $|\frac{1}{2}x|-2=|\frac{1}{2}x+2|-1 \Longrightarrow \underbrace{|\frac{1}{2}x|-|\frac{1}{2}x+2|}_{y}=\underset{\tilde{y}}{1}$

$$y=\begin{cases} x\geq 0\;;\; (\frac{1}{2}x)-(\frac{1}{2}x+2)=-2 \\ -4<x<0\;;\; -(\frac{1}{2}x)-(\frac{1}{2}x+2)=-x-2 \\ x\leq -4\;;\; -(\frac{1}{2}x)+(\frac{1}{2}x+2)=2 \end{cases}$$

$\tilde{y}=1 \quad -x-2=1 \Rightarrow x=-3$

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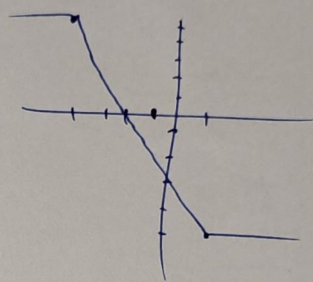
۴. ۱,۵

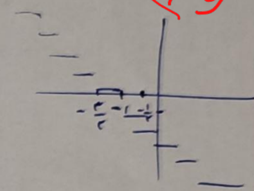
الف) $f(x)=[|1-x|-|x+4|]$

$x\geq 1\;;\; (x-1)-(x+4)=-5$

$-4\leq x<1 \quad -(x-1)-(x+4)=-2x-3$

$x\leq -4 \quad -(x-1)+(x+4)=5$





$=\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

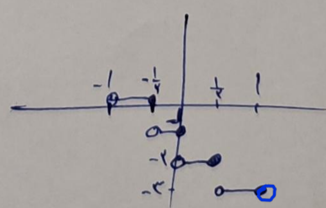
ب) $f(x)=\frac{1}{x}-[\frac{x+1}{x}]=\frac{1}{x}-[1+\frac{1}{x}]=\underbrace{\frac{1}{x}-[\frac{1}{x}]}_{0\leq\;<1}\cancel{+}1 \longrightarrow R=[-1,0)$

$0\leq\frac{1}{x}-[\frac{1}{x}]<1 \Rightarrow 1\leq\frac{1}{x}-[\frac{1}{x}]+1<2 \Rightarrow R_f=\cancel{[1,2)}$

۲- ۱

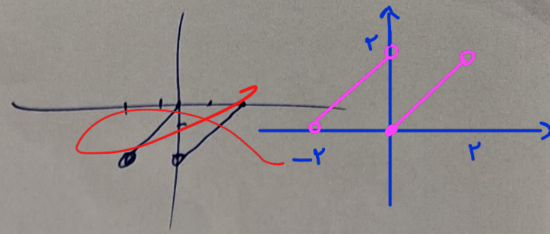
الف) $y=[-2x]-1 \quad x\in(-1,1]$

$$\begin{cases} -1<x\leq-\frac{1}{2}\;;\; 1-1=0 \\ -\frac{1}{2}<x\leq 0\;;\; 0-1=-1 \\ 0<x\leq\frac{1}{2}\;;\; -1-1=-2 \\ \frac{1}{2}<x\leq 1\;;\; -2-1=-3 \end{cases}$$



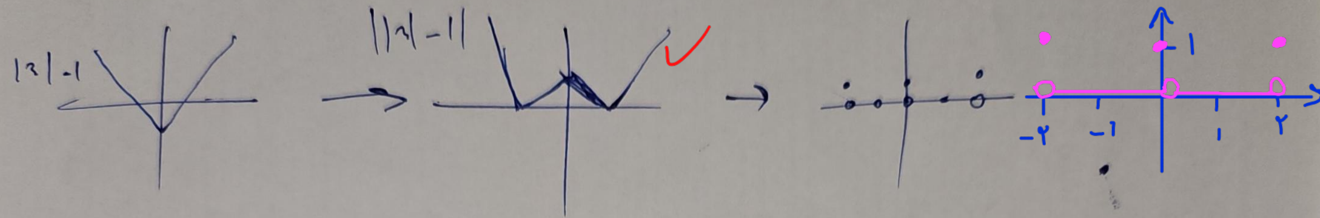
ب) $x - 2[\frac{x}{2}]$ $\quad x\in(-2,2)$

$2(\frac{x}{2}) - 2[\frac{x}{2}] = 2(\frac{x}{2} - [\frac{x}{2}])$

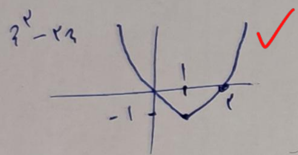


سوال ۸ ①

الف) $y=[||x|-1|]$ $\quad x\in[-2,2]$

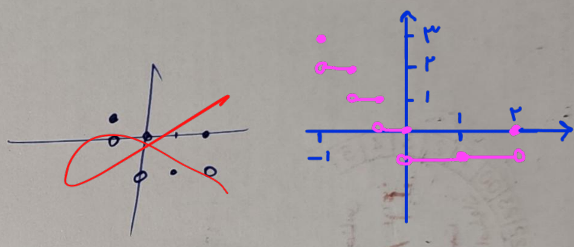


ب) $y=[x^2-2x]$ $\quad x\in[-1,2]$



$x_s = -\frac{b}{2a} = 1$

$y_s = -1$



سوال ۹ 1,5

$a+[b]=4,2 \longrightarrow [a]+[b]=4 \Rightarrow 2[b]=6 \Rightarrow [b]=3$

$b-[a]=2,4 \longrightarrow [b]-[a]=2$ $\quad [a]+3=4 \Rightarrow [a]=1$

$a=1+0,2=1,2$

$b=\cancel{\ldots}2,4$ $\quad b-[\cancel{a}]=2,4 \longrightarrow b=3,4$

$\Rightarrow a+b=\cancel{4,8}\ \boxed{4,6}$

سوال ۱۰ ②

$(1+\sqrt2)^6+(1-\sqrt2)^6=198 \qquad [(1+\sqrt2)^6]=?$

$(1+\sqrt2)^6=t \qquad t+\frac{1}{t}=198 \Rightarrow t^2-198t+1=0 \Rightarrow t=\frac{198\pm\sqrt{198^2-4}}{2} \Rightarrow t=\frac{198\pm140\sqrt2}{2}$

$\frac{1}{t}=\frac{1}{(1+\sqrt2)^6}=\frac{(\sqrt2-1)^6}{(2-1)^6}=(\sqrt2-1)^6$

$=99\pm70\sqrt2$

$\longrightarrow \sqrt2\simeq1,41 \Rightarrow 99+70\times1,41=197,7$

$[197,7]=197$ ✓

۲ب

$$\left|\frac{x-2}{2x+1}\right| > 1 \qquad x \neq \frac{-1}{2}$$

$$|x-2| > |2x+1| \xrightarrow{\text{توان ۲}} x^2-4x+4 > 4x^2+4x+1$$

$$3x^2+8x-3<0 \longrightarrow \begin{array}{c|c|c} & -3 & \frac{1}{3} \\ \hline + & - & + \end{array} \qquad \left(-3, \frac{1}{3}\right) - \left\{\frac{-1}{2}\right\}$$