علی ربانی

پاسخنامه تکلیف شماره :

کلاس یازدهم

۱- $\{2\}$

$\frac{1}{x-1} = |x-1|$

$x > 1 \Rightarrow x^2 - 2x + 1 = 1$

$x^2 - 2x = 0 \quad x(x-2) = \begin{cases} 0 & \times \\ 2 & \checkmark \end{cases}$

$x < 1 \Rightarrow (x-1)(-x+1) = -x^2 + 2x - 1 = 1 \Rightarrow x^2 - 2x + 2 = 0 \xrightarrow{\Delta < 0}$ فاقد جواب

۲-

$\left|\frac{2x-1}{x}\right| < 2 \Rightarrow -2 < \frac{2x-1}{x} < 2 \Rightarrow -2x < 2x - 1 \quad 1 < 4x \quad \boxed{\frac{1}{4} < x} \quad = [\frac{1}{4}, +\infty)$

$\rightarrow \left|\frac{x-2}{2x+1}\right| > 1 \quad 1 < \frac{x-2}{2x+1} \quad 0 < \frac{x-2-2x-1}{2x+1} \Rightarrow 0 < \frac{-x-3}{2x+1} \quad (-3, -\frac{1}{2})$

$-1 > \frac{x-2}{2x+1} \quad 0 > \frac{x-2+2x+1}{2x+1} \quad 0 > \frac{3x-1}{2x+1} \quad (-\frac{1}{2}, \frac{1}{3})$

$(-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{3}) = \boxed{(-3, \frac{1}{3}) - \{-\frac{1}{2}\}}$

۳-

$x^2 < |x+6| \rightarrow \begin{cases} x > -6 & x^2 < x + 6 & x^2 - x - 6 < 0 & (x-3)(x+2) < 0 \\ x < -6 & x^2 < -x - 6 & x^2 + x + 6 < 0 \xrightarrow{\Delta < 0} & \text{فاقد جواب } \times \end{cases}$

$-2 \quad 3$

$|x - \alpha| < \beta \qquad \alpha = \frac{-2+3}{2} = \frac{1}{2}$

$|x - \frac{1}{2}| < \beta \Rightarrow \boxed{\alpha = \frac{1}{2}}$

۴-

$y = |x+1| - |2x| + |x-2| \qquad max = 3, \quad min = -1 \qquad 3 - 1 = \boxed{2}$

$-1 \quad 0 \quad 2$

$-x-1+2x-x+2 = 3 \quad | \quad 1 \quad | \quad 2x+2 \quad 3 \quad | \quad -2x+3 \quad | \quad -1 \quad | \quad -1$

۵- -3

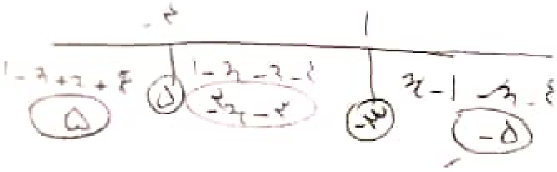
$y = |\frac{1}{2}x| - 2 \Rightarrow y' = |\frac{1}{2}x + 2| - 2 + 1$

$y' = |\frac{1}{2}x + 2| - 1 \qquad y' = y \Rightarrow |\frac{1}{2}x| - 1 = |\frac{1}{2}x + 2| - 1$

ریشه $= 0, -4$

$\begin{cases} 0 \le x & \frac{1}{2}x - 1 = \frac{1}{2}x + 2 & -1 \ne 2 & \times \\ -4 \le x \le 0 & -\frac{1}{2}x - 1 = \frac{1}{2}x + 2 \Rightarrow \boxed{x = -3} \\ x \le -4 & -\frac{1}{2}x - 1 = -\frac{1}{2}x - 2 & -1 \ne -2 & \times \end{cases}$

۶- الف) $f(x) = [\,|1-x| - |x+4|\,]$



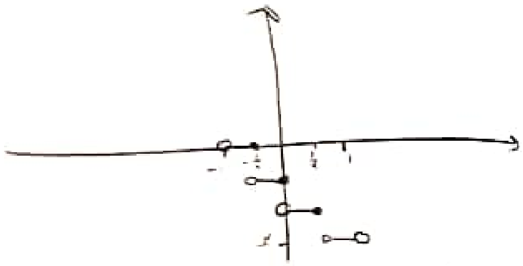
$\Rightarrow R_f = \underbrace{[-5, -4, -3, \dots, 3, 4, 5]}_{\in \mathbb{Z}}$

ب) $\frac{1}{x} - [1 + \frac{1}{x}] = \frac{1}{x} - 1 - [\frac{1}{x}] = \begin{cases} x \in \mathbb{Z} = \frac{1}{x} - 1 \\ \end{cases}$

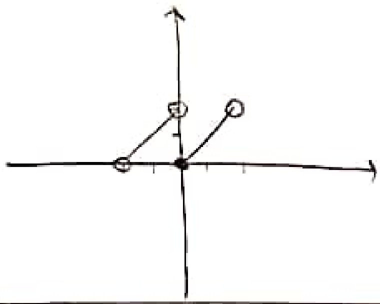
$-1 \leq \frac{1}{x} - 1 \leq 0$ $\qquad R_f = [-1, 0]$

۷- الف) $y = [-2x] - 1 \quad \xrightarrow{x \in (-1,1)}$ $\begin{cases} -2 < 2x \leq -1 \Rightarrow [2x] = -1 \quad [-2x] = 1 \Rightarrow y = 0 \\ -1 < 2x \leq 0 \Rightarrow [2x] = 0 \quad [-2x] = 0 \Rightarrow y = -1 \\ 0 < 2x \leq 1 \Rightarrow [2x] = 1 \quad [-2x] = -1 \Rightarrow y = -2 \\ 1 < 2x < 2 \Rightarrow [2x] = 2 \quad [-2x] = -2 \Rightarrow y = -3 \end{cases}$

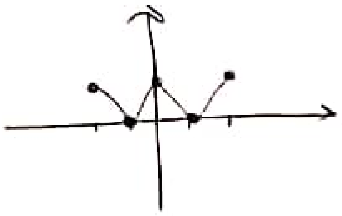
$x \in (-1, 1) \Rightarrow 2x (-2, 2)$



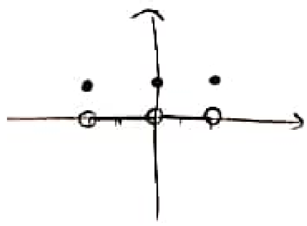
ب) $y = x - 2[\frac{x}{2}] \quad \xrightarrow{x \in (-2,2)}$ $\begin{cases} -1 < \frac{x}{2} < 0 \Rightarrow [\frac{x}{2}] = -1 \Rightarrow y = x + 2 \quad x \in (-2, 0) \\ 0 \leq \frac{x}{2} < 1 \Rightarrow [\frac{x}{2}] = 0 \Rightarrow y = x \quad x \in (0, 2) \end{cases}$



۸- الف) $y = [\,||x| - 1|\,] \quad x \in [-2, 2]$

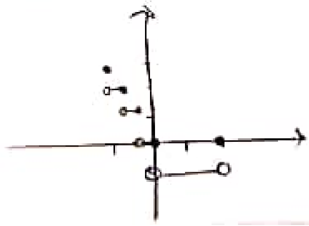


$\xrightarrow{[\ ]}$



ب) $y = [x^2 - 2x] \quad x \in [-1, 3]$

$\xrightarrow{[\ ]}$



۹-

$a + [b] = ۴,۲ \qquad b = x,۴ \qquad b = ۳,۴$

$b - [a] = ۲,۴ \qquad a = y,۲ \qquad a = ۱,۲$

$۱,۲ + ۳,۴ = \boxed{۴,۶}$

$x - y = ۲ \qquad x + y = ۴$

$۲x = ۶ \qquad \boxed{x = ۳} \qquad \boxed{y = ۱}$

۱۰-

$(۱+\sqrt{۲})^۶ + (۱-\sqrt{۲})^۶ = ۱۹۸ \qquad (۱+\sqrt{۲})^۶ = ۱۹۸ - (۱-\sqrt{۲})^۶$

$۰<|۱-\sqrt{۲}|<۱$

$(۱+\sqrt{۲})^۶ = ۱۹۷,\ldots \Rightarrow [(۱+\sqrt{۲})^۶] = \boxed{۱۹۷}$

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