

«امیر علی وحید» «یازدهم گنبد»

$$\frac{1}{x-1} \rightarrow x \neq 1 \quad \frac{1}{x-1} = |x-1| \rightarrow \begin{cases} x > 1 \rightarrow \frac{1}{x-1} = x-1 \rightarrow x^2 + 1 - 2x = x \rightarrow x^2 - 2x = 0 \\ x < 1 \rightarrow \frac{1}{x-1} = 1-x \rightarrow -x^2 + 1 - 2x = 0 \end{cases}$$

$$x(x-2) = 0 \rightarrow x = 0 \text{ or } x = 2$$

$$x^2 + 2x + 2 = 0 \rightarrow \Delta < 0$$

الف) $\left| \frac{x-1}{x} \right| < 2 \rightarrow -2 < \frac{x-1}{x} < 2 \rightarrow \frac{-2x < x-1 < 2x}{x} \rightarrow 0 < 1 \checkmark$

ب) $\frac{x-2}{x+1} > 1, \frac{x-2}{x+1} < -1$

$$\frac{x-2}{x+1} > 1 \rightarrow \frac{x-2-x-1}{x+1} > 0 \rightarrow \frac{-2x-3}{x+1} > 0 \rightarrow \frac{-2 \quad -\frac{3}{2}}{-1 \quad 0} \rightarrow (-\frac{3}{2}, -1)$$

$$\frac{x-2}{x+1} < -1 \rightarrow \frac{x-2+x+1}{x+1} < 0 \rightarrow \frac{2x-1}{x+1} < 0 \rightarrow \frac{\frac{1}{2} \quad 1}{-1 \quad 0} \rightarrow (-1, \frac{1}{2})$$

$$\Rightarrow (-\frac{3}{2}, \frac{1}{2}) - \{-1\}$$

$$x^2 < x+4 \rightarrow x^2 - x - 4 < 0 \rightarrow (x-\frac{1}{2})(x+\frac{7}{2}) < 0 \rightarrow \frac{-\frac{1}{2} \quad \frac{7}{2}}{-1 \quad 0} \rightarrow (-\frac{7}{2}, \frac{1}{2})$$

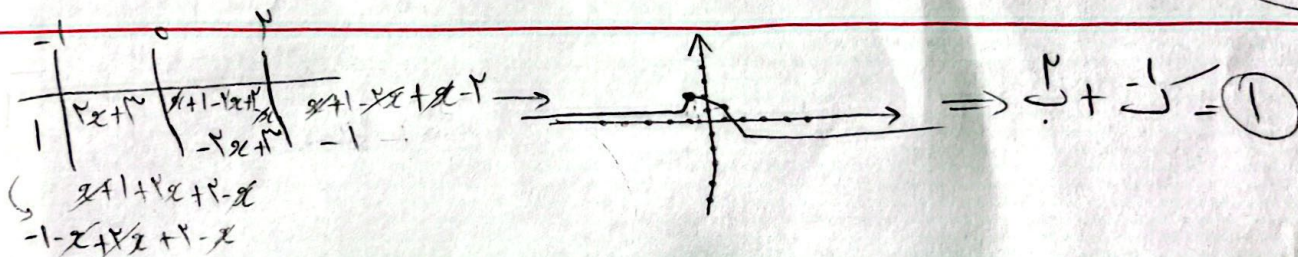
$$-x^2 > x+4 \rightarrow 0 > x^2 + x + 4 \rightarrow \Delta < 0$$

$$-\beta < x - \alpha < \beta$$

$$-\beta + \alpha < x < \beta + \alpha \rightarrow \alpha - \beta = -2$$

$$\alpha + \beta = 3$$

$$2\alpha = 1 \rightarrow \alpha = \frac{1}{2}, \beta = \frac{5}{2}$$



$$x = \left| \frac{1}{2}x - 2 \right| - 2 + 1 \Rightarrow \left| \frac{1}{2}x - 2 \right| - 1 = x \rightarrow \left| \frac{1}{2}x - 2 \right| - x = 1$$

$$x^2 - 12x + 24 = 0$$

$$(x-3)(x-8) \Rightarrow x = 3, 8$$

$$x = \frac{1}{2}x$$

$$\left| \frac{1}{2}x - 2 \right| + 1 = \left| \frac{1}{2}x \right|$$

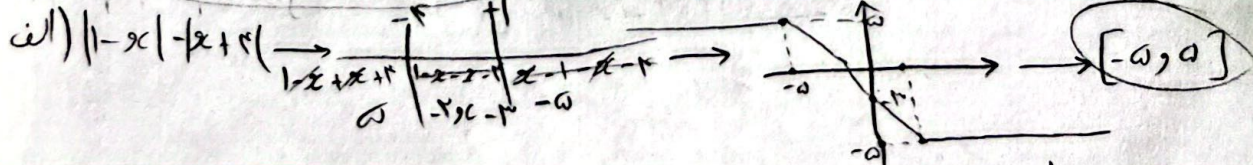
$$\frac{1}{2}x + 1 - 2x + 1 + 2 \left| \frac{1}{2}x - 2 \right| = \frac{1}{2}x$$

$$2 \left| \frac{1}{2}x - 2 \right| = 2x - 4 \leftarrow \omega - 2x + 2 \left| \frac{1}{2}x - 2 \right| = 0$$

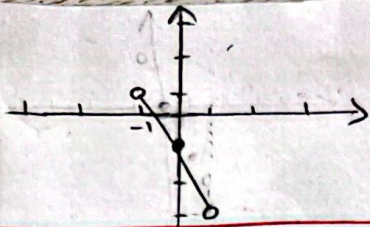
$$2 \left(\frac{1}{2}x^2 + 1 - 2x \right) = x^2 + 1 - 4x = x^2 + 1 - 4x$$

ب) $\frac{1}{x} - \left[\frac{1}{x} \right] + 1$

$$1 \leq \frac{1}{x} - \left[\frac{1}{x} \right] + 1 < 2$$

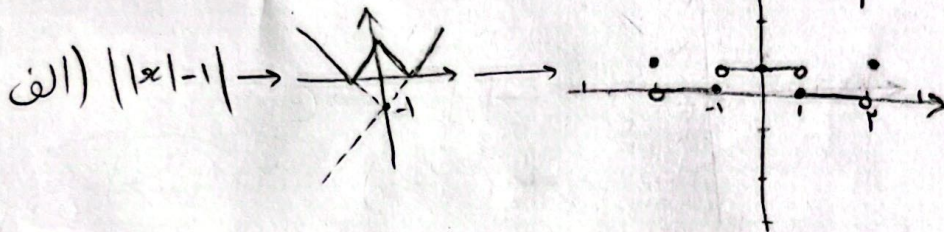
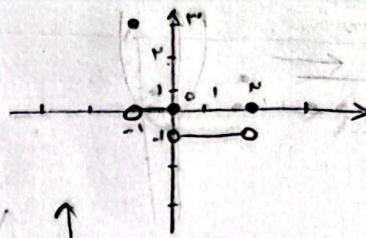


ب) $x - 2 \left[\frac{x}{2} \right] \rightarrow 2 \left(\frac{x}{2} - \left[\frac{x}{2} \right] \right) \rightarrow 0 \leq \frac{x}{2} - \left[\frac{x}{2} \right] < 1$



(الف)

ب) $x^2 - 2x \rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 - x \rightarrow x = -1$
 $x(x-2) \Rightarrow x=0, x=2$



$a \rightarrow$ عدد صحیح نیست $\rightarrow b = 2,4 + [a] \rightarrow a + [2,4 + [a]] = 2,2$
 $a + [0,4 + [a]] = 2,2$
 بدون این ایهی نداریم $\rightarrow a + [[a]] = 2,2 \rightarrow a + [a] = 2,2$
 بدون این برآیند هم تأییدی نداریم $\rightarrow a = 1,2$
 $a + b = 4,4 \leftarrow b = 2,4 \leftarrow a = 1,2$

$(1+\sqrt{2})^8 + (1-\sqrt{2})^8 = 198 \rightarrow ((1+\sqrt{2})^4 + (1-\sqrt{2})^4)^2 = (1+\sqrt{2})^8 + (1-\sqrt{2})^8 + 2$
 $(1+\sqrt{2})^4 = 14 + 14\sqrt{2} + 14 + 14\sqrt{2} = 28 + 28\sqrt{2}$
 $(1-\sqrt{2})^4 = 14 - 14\sqrt{2} + 14 - 14\sqrt{2} = 28 - 28\sqrt{2}$
 $(1+\sqrt{2})^4 + (1-\sqrt{2})^4 = 56$
 $(56)^2 = 3136$
 $3136 - 2 = 3134$
 $3134 = 198 \times 16$
 $198 \times 16 = 3168$
 $3134 - 3168 = -34$
 $-34 = -19 \times 2$
 $-19 \times 2 = -38$
 $-38 + 4 = -34$
 $-34 + 34 = 0$
 $0 = 0$
 $19 \checkmark$