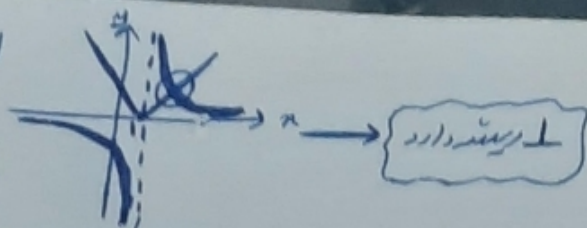


1) $\frac{1}{n-1} = |n-1|$



2) الف) $|\frac{2n-1}{n}| < 2 \Rightarrow |\frac{2n}{n} - \frac{1}{n}| < 2 \Rightarrow |2 - \frac{1}{n}| < 2 \Rightarrow -2 < 2 - \frac{1}{n} < 2 \Rightarrow -4 < -\frac{1}{n} < 0$

$\Rightarrow 0 < \frac{1}{n} < 4 \Rightarrow \frac{1}{4} < n < +\infty$ $(\frac{1}{4}, +\infty)$

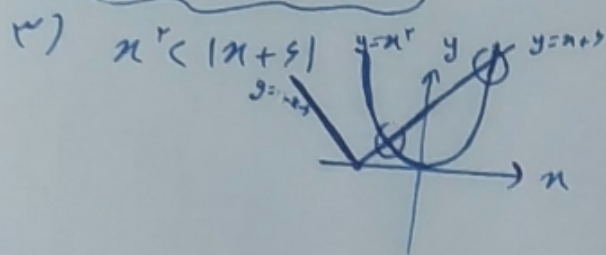
ب) $|\frac{n-2}{n+1}| > 1 \Rightarrow \begin{cases} \frac{n-2}{n+1} > 1 \Rightarrow \frac{n-2}{n+1} - 1 > 0 \Rightarrow \frac{-n-3}{n+1} > 0 \Rightarrow \frac{n+3}{n+1} < 0 \Rightarrow \frac{n}{n+1} < -\frac{3}{n+1} \Rightarrow \frac{n}{n+1} < -\frac{3}{n+1} \Rightarrow n < -3 \end{cases}$

$\frac{n}{n+1} < -\frac{3}{n+1} \Rightarrow n < -3$ $(-\infty, -3)$

ج) $|\frac{n-2}{n+1}| < 1 \Rightarrow \begin{cases} \frac{n-2}{n+1} < 1 \Rightarrow \frac{n-2}{n+1} - 1 < 0 \Rightarrow \frac{-n-3}{n+1} < 0 \Rightarrow \frac{n+3}{n+1} > 0 \Rightarrow \frac{n}{n+1} > -\frac{3}{n+1} \Rightarrow \frac{n}{n+1} > -\frac{3}{n+1} \Rightarrow n > -3 \end{cases}$

$\frac{n}{n+1} > -\frac{3}{n+1} \Rightarrow n > -3$ $(-3, +\infty)$

د) $U(I) \Rightarrow (-3, -\frac{1}{4}) \cup (-\frac{1}{4}, \frac{1}{4})$

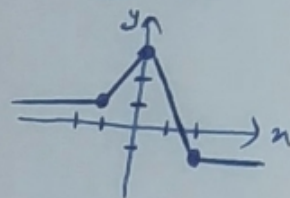


ه) $n^2 < |n+6| \Rightarrow n^2 - n - 6 < 0 \Rightarrow (n-3)(n+2) < 0 \Rightarrow \begin{cases} n_1 = -2 \\ n_2 = 3 \end{cases}$ $(-2, 3)$

مطابق شکل و (I) $\alpha = \frac{3+(-2)}{2} = \frac{1}{2}$
 $\beta = \frac{1-(-2)}{2} = \frac{3}{2}$

$\Rightarrow |n - \frac{1}{2}| < \frac{3}{2}$

$\alpha = \frac{1}{2}$



و) $y = |x+1| - |2x| + |x-2| \Rightarrow y = \begin{cases} x+1-2x+x-2 = -1 & x \geq 2 \\ x+1-2x-x+2 = -2x+3 & 0 \leq x < 2 \\ x+1+2x-x+2 = 2x+3 & -1 \leq x < 0 \\ -x+1-2x-x+2 = -4x+3 & x < -1 \end{cases}$

$\Rightarrow \begin{cases} \max(y) = 3 \\ \min(y) = -1 \end{cases} \Rightarrow 3 + (-1) = \boxed{2}$

ز) $y_r = |\frac{1}{r}(n+r)| - r + 1 = |\frac{1}{r}n + 1| - r + 1$
 $y_l = |\frac{1}{r}n| - r$

$\Rightarrow |x| = |n+r| + r \Rightarrow \begin{cases} n = n+r & n \geq 0 \\ -n = n+r & n < 0 \end{cases}$

$n = -r$

ح) الف) $f(n) = [||n|-|n+r||] \rightarrow \begin{cases} f(n) = [n-1-n-r] = [-d] = -d & n \geq 1 \\ f(n) = [1-n-n-r] = [-2n-r] & -r < n < 1 \\ f(n) = [1-n+n+r] = [d] = d & n \leq -r \end{cases}$

$\Rightarrow d \leq [-2n] - r \leq d$ (I)

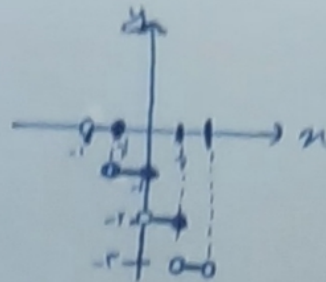
$\Rightarrow R_f = \{n \mid n \in \mathbb{Z}, n \in [-d, d]\}$

ب) $f(n) = \frac{1}{n} - [\frac{n+1}{n}] = \frac{1}{n} - [\frac{n}{n} + \frac{1}{n}] = \frac{1}{n} - [1 + \frac{1}{n}] = -1$

$\Rightarrow R_f = \{-1, -2\}$

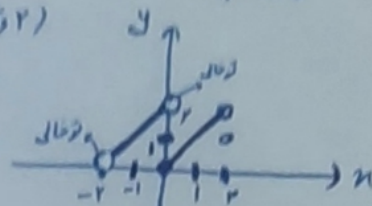
$t - [t] \rightarrow \begin{cases} t \in \mathbb{Z} \Rightarrow t - [t] = 0 \\ t \notin \mathbb{Z} \Rightarrow t - [t] = -1 \end{cases}$

v) $y = [-rn] - 1 \quad n \in (-b, 1)$

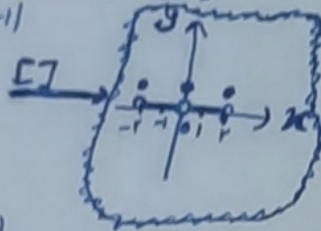
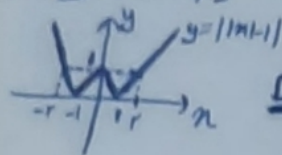


یا در حالت کلیت = یازدهم پسر A

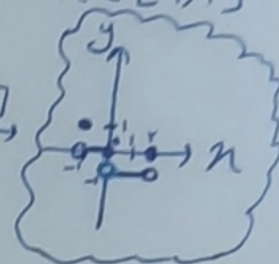
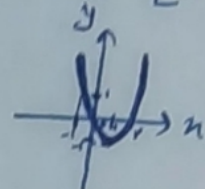
—) $y = n - r[n/r] \quad n \in (-b, r)$



٨) الف) $y = [|n| - 1] \quad n \in [-r, r]$



ب) $y = [n^2 - rn] \quad n \in [-1, r]$



9) $a + [b] = r, r \xRightarrow{[]} [a + [b]] = [r, r] \Rightarrow [a] + [b] = r$
 $b - [a] = r, r \xRightarrow{[]} [b - [a]] = [r, r] \Rightarrow [b] - [a] = r$

$a + [b] = r, r \xRightarrow{[b]=r} a = 1, r$
 $b - [a] = r, r \xRightarrow{[a]=1} b = r, r$
 $\Rightarrow a + b = 1, r + r, r = \boxed{r, r}$

١٥) $(1 + \sqrt{2})^5 + (1 - \sqrt{2})^5 = 19\lambda$

$1 - \sqrt{2} \leq -0.41 \Rightarrow -1 < (1 - \sqrt{2})^5 < 0$
 $\Rightarrow -1 < -(1 - \sqrt{2})^5 < 0$
 $\Rightarrow \lfloor -(1 - \sqrt{2})^5 \rfloor = -1 \quad (I)$

$(1 + \sqrt{2})^5 = 19\lambda - (1 - \sqrt{2})^5$
 $\Rightarrow \lfloor (1 + \sqrt{2})^5 \rfloor = \lfloor 19\lambda - (1 - \sqrt{2})^5 \rfloor = 19\lambda + \lfloor -(1 - \sqrt{2})^5 \rfloor$
 $(I) \Rightarrow \lfloor (1 + \sqrt{2})^5 \rfloor = 19\lambda + (-1) = \boxed{19\lambda - 1}$