شایان رومی زاده

۱۶٫۲۵

$x-1 \geq 0 \rightarrow x > 1 \Rightarrow \frac{1}{x-1} = x-1 \rightarrow x^2 - 2x + 1 = 1 \rightarrow x(x-2) = 0$ ↙ $x=0$ غ‌ق‌ق ، $x=2$ ق‌ق

$x-1 = 0$ غ‌ق‌ق

$x-1 < 0 \rightarrow \frac{1}{x-1} = -x+1 \Rightarrow -x^2 - 1 + 2x = 1 \rightarrow x^2 - 2x + 2 = 0 \xrightarrow{\Delta<0}$ ریشه حقیقی ندارد

$\rightarrow x = 2$ ✓ ۱ ریشه

② ۱

ریشه‌ها $= \frac{1}{2}, 0$

0		$\frac{1}{2}$	
$\frac{2x-1}{x}$ ①	$\frac{1-2x}{x}$ ②	$\frac{2x-1}{x}$ ③	

$0 \rightarrow$ ریشه مخرج‌ها

①: $\frac{2x-1}{x} < 2 \rightarrow$ ✓ $(-\infty, 0)$

②: $\frac{1-2x}{x} < 2 \rightarrow 1 < 4x \rightarrow \boxed{\frac{1}{4} < x < \frac{1}{2}}$

③: $\frac{2x-1}{x} < 2 \rightarrow$ ✓ $[\frac{1}{2}, +\infty)$

$\cup \rightarrow \boxed{(-\infty, 0) \cup (\frac{1}{4}, +\infty)}$ ✓

①

$\frac{x-2}{2x+1} \rightarrow$ ریشه‌ها $= 2, -\frac{1}{2}$

$-\frac{1}{2}$		2	
$\frac{x-2}{2x+1}$ ①	$\frac{2-x}{2x+1}$ ②	$\frac{x-2}{2x+1}$ ③	

①: $\frac{x-2}{2x+1} > 1 \rightarrow x-2 > 2x+1 \rightarrow x < -3 \rightarrow (-\infty, -3)$ ①

②: $\frac{2-x}{2x+1} > 1 \rightarrow 2-x > 2x+1 \rightarrow x < \frac{1}{3} \rightarrow (-\frac{1}{2}, \frac{1}{3})$ ②

③: $\frac{x-2}{2x+1} > 1 \rightarrow x-2 > 2x+1 \rightarrow x < -3 \rightarrow \varnothing$ ③

$(-3, -\frac{1}{2})$

① ∪ ② ∪ ③ $= \boxed{(\cancel{-\infty, -3}) \cup (-\frac{1}{2}, \frac{1}{3})}$

$x+6 \geq 0 \rightarrow x^2 < x+6 \rightarrow x^2 - x - 6 < 0 \rightarrow (x+2)(x-3) < 0 \rightarrow (-2, 3)$

$x+6 < 0 \rightarrow x^2 < -x-6 \rightarrow x^2 + x + 6 < 0 \xrightarrow{\Delta<0}$ ریشه حقیقی ندارد

$\frac{(-2)+(3)}{2} = \frac{1}{2} \rightarrow = |x - \frac{1}{2}| < \beta \xrightarrow{x=3} \beta = 2.5 \rightarrow |x - \frac{1}{2}| < \frac{5}{2}$

$\rightarrow \boxed{\alpha = \frac{1}{2}}$ ✓

② ۳

ریشه مطلق‌ها $= -1, 0, 2$

-1	0	2	
$-x-1+2x-x+2 = 1$	$x+1+2x-x+2 = 2x+3$	$x+1-2x-x+2 = -2x+3$	$x+1-2x+x-2 = -1$
1	$(1, 3)$	$(-1, 3)$ ، 3	-1

$\min = -1$ ✓

$\max = 3$

$3 + (-1) = 2$ ✓

② ۴

تغییرات فقط روی خود x اجرا میشه

$|\frac{1}{2}x| - 2 \xrightarrow{\text{۴ واحد به چپ}} |\frac{1}{2}(x+4)| - 2 \xrightarrow{\text{۱ واحد بالا}} |\frac{1}{2}x + 4| - 1$

①

$|\frac{1}{2}x| - 2 = |\frac{1}{2}x + 4| - 1 \xrightarrow{\times 2}$

$-8 \leq x < 0 \rightarrow -\frac{x}{2} - 2 = +\frac{1}{2}x + 4 - 1$ غ‌ق‌ق

$\rightarrow \boxed{-5 = x}$

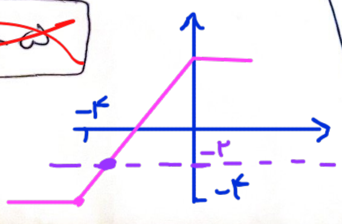
$0 \leq x \Rightarrow \frac{1}{2}x - 2 = \frac{1}{2}x + 4 - 1 \rightarrow$ غ‌ق

$x < -8 \rightarrow -\frac{x}{2} - 2 = -\frac{x}{2} - 4 - 1 \rightarrow$ غ‌ق

۵

تابع آبشاری $|x+4| - |x| = -2$

$\boxed{\cancel{x = -5}}$



$-4 < x < 0$

$x + 4 + x = -2 \rightarrow \boxed{x = -3}$

$1 \le x \rightarrow [x-1-x-4] = [-5] = -5$ ①

$-4 \le x \le 1 \rightarrow [1-x-x-4] = [-2x]-3$

$x \le -4 \rightarrow [-x+1+x+4] = [5] = 5$ ③

$x=-4 \rightarrow y = 8-3=5$
$x=1 \rightarrow y=-5$
② $[-5,5]$

① ∪ ② ∪ ③ $= [-5,5] \rightarrow \boxed{R=[-5,5]}$ ✓ $\rightarrow \{-5,-4,\dots,5\}$

چون براکت داریم، برد فقط شامل اعداد صحیح این بازه می‌باشد

۰٫۷۵ 6

~~$-1 \le x < 0$~~
~~$0 \le x < 1 \rightarrow$~~
الف)

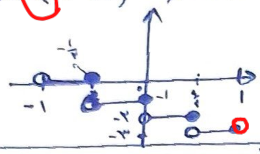
$-1<x<1 \rightarrow 2>-2x>-2 \rightarrow$

$-2<-2x\le -1 \rightarrow [-2x]=-2,\ \frac12 \le x<1 \rightarrow y=-2$

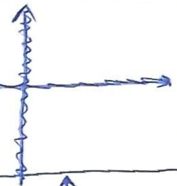
$-1\le -2x<0 \rightarrow [-2x]=-1,\ 0<x\le\frac12 \rightarrow y=-2$

$0\le -2x<1 \rightarrow [-2x]=0,\ -\frac12<x\le 0 \rightarrow y=-1$

$1\le -2x<2 \rightarrow [-2x]=1,\ -1<x\le -\frac12 \rightarrow y=0$



دقت!

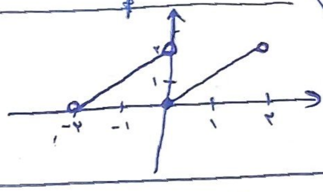


۰٫۷۵ 7

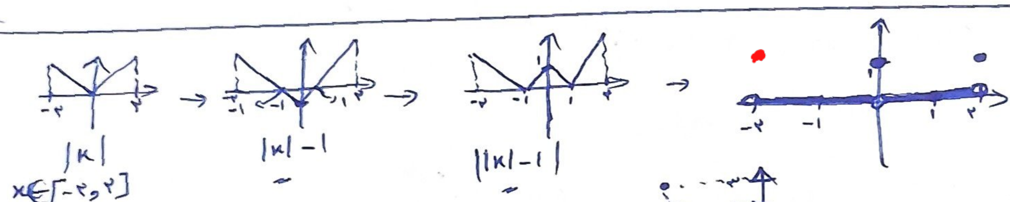
$-2<x<2 \rightarrow -1<\frac{x}{2}<1 \longrightarrow$

$-1<\frac{x}{2}<0 \rightarrow [\frac{x}{2}]=-1,\ y=x+2,\ -2<x<0$

$0\le \frac{x}{2}<1 \rightarrow [\frac{x}{2}]=0,\ y=x,\ 0\le x<2$

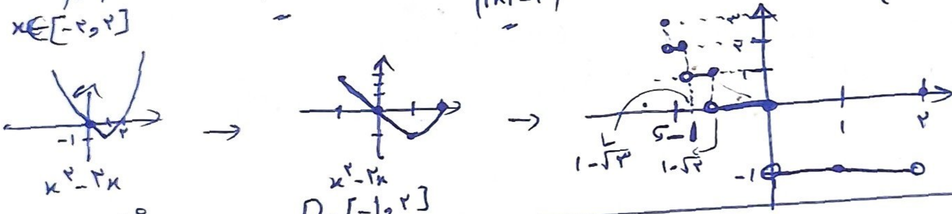


✓



$|x|$, $x\in[-2,2]$ → $|x|-1$ → $||x|-1|$ → $[||x|-1|]$, $x\in[-2,2]$

۱٫۷۵ 8



x^2-2x
$= x(x-2)<0$

x^2-2x
$D=[-1,2]$

✓

~~$\rightarrow [a]+0.2+[b]=4.2$~~

$[a+[b]]=[4.2]$, $[b-[a]]=[2.4]$

$\rightarrow [a]+[b]=4$, $[b]-[a]=2 \rightarrow 2[b]=6 \rightarrow [b]=3$, $[a]=1$

$a+[b]=4.2 \rightarrow a=4.2-[b]=1.2$ ✓

$b-[a]=2.4 \rightarrow b=2.4+1=3.4$ ✓

$\rightarrow a+b=3.4+1.2=\boxed{4.6}$ ✓

۲ 9

$(1+\sqrt2)^2=3+2\sqrt2$, $(1-\sqrt2)^2=3-2\sqrt2 \Rightarrow \left((1+\sqrt2)^2\right)^3+\left((1-\sqrt2)^2\right)^3=(3+2\sqrt2)^3+(3-2\sqrt2)^3=$

$(3+2\sqrt2+3-2\sqrt2)^3-3(3+2\sqrt2)(3-2\sqrt2)(3+2\sqrt2+3-2\sqrt2)$

$=6^3-3(1)(6)=198$

~~$(1+\sqrt2)^6+(1-\sqrt2)^6=(1+\sqrt2)^6+(1-1.4)^6$~~
~~$\sqrt2 = 1.4$~~

۲ 10

$(1+\sqrt2)^6+(1-\sqrt2)^6=198 \xrightarrow{\sqrt2\simeq1.4} (1+\sqrt2)^6=198-\underbrace{(-0.4)^6}_{\text{عدد مثبت کوچکتر از یک}} < 198$, $197<198-(-0.4)^6$

$\rightarrow 197<(1+\sqrt2)^6<198 \longrightarrow \boxed{[1+\sqrt2]^6=197}$ ✓

۲-

$$-2 < \frac{2x-1}{x} < 2 \rightarrow -2 < 2-\frac{1}{x} < 2 \rightarrow -4 < \frac{-1}{x} < 0$$

$$\begin{cases} \frac{-1}{x} > -4 \rightarrow x > \frac{1}{4} \\ \frac{-1}{x} < 0 \rightarrow x > 0 \end{cases} \xrightarrow{\cap} \boxed{x > \frac{1}{4}}$$

$$\left|\frac{x-2}{2x+1}\right| > 1 \qquad x \neq \frac{-1}{2}$$

$$|x-2| > |2x+1| \xrightarrow{\text{توان ۲}} x^2-4x+4 > 4x^2+4x+1$$

$$3x^2+8x-3 < 0 \longrightarrow \begin{array}{c|c|c} & -3 \quad & \quad \frac{1}{3} \\ \hline + & - & + \end{array} \quad \left(-3, \frac{1}{3}\right) - \left\{\frac{-1}{2}\right\}$$

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۴ ب

$$\frac{1}{x} - \left[1+\frac{1}{x}\right] = \underbrace{\frac{1}{x} - \left[\frac{1}{x}\right]}_{0 \leq \quad < 1} - 1 \xrightarrow{-1} R = [-1, 0)$$