

۱۷

دقت:

$$\frac{1}{x-1} = |n-1|$$

$x > 1$

$$\frac{1}{x-1} = x-1$$

$$x \neq 1 \rightarrow (x-1)^2 = 1$$

$$x^2 - 2x + 1 = 1 \rightarrow x^2 - 2x = 0$$

$$x(x-2) = 0 \rightarrow x = 0, 2$$

۲ جواب دارد

$x < 1$

$$\frac{1}{x-1} = -x+1$$

$$x \neq 1 \rightarrow -(x-1)^2 = 1$$

$$x^2 - 2x + 1 = -1 \rightarrow x^2 - 2x + 2 = 0$$

$$\Delta = (-2)^2 - 4(1)(2) = -4 < 0$$

خواب ندارد

در مجموع ۲ جواب دارد

الف)  $\left| \frac{x-1}{x} \right| < 2 \rightarrow \frac{|x-1|}{|x|} < 2 \rightarrow |x-1| < 2|x|$  (سوال ۲)

$$\rightarrow (|x-1|)^2 < (2|x|)^2 \Rightarrow x^2 - 2x + 1 < 4x^2 \Rightarrow 3x^2 + 2x - 1 > 0 \Rightarrow x > \frac{1}{3}$$
 (۷۵)

ب)  $\left| \frac{x-2}{x+1} \right| > 1 \rightarrow \frac{|x-2|}{|x+1|} > 1 \rightarrow |x-2| > |x+1|$

$$\rightarrow (|x-2|)^2 > (|x+1|)^2 \rightarrow x^2 - 4x + 4 > x^2 + 2x + 1 \rightarrow -6x + 3 > 0 \rightarrow x < \frac{1}{2}$$

$$\left( -\frac{1}{2}, \frac{1}{3} \right) \cup \left( \frac{1}{2}, 1 \right)$$

پیش فرض

$$\frac{x}{|x|} = \frac{-3}{|x|} \Rightarrow \frac{1}{x} = -3 \Rightarrow x = -\frac{1}{3}$$

$$D_f = \left( -\frac{1}{3}, -\frac{1}{2} \right) \cup \left( \frac{1}{2}, 1 \right)$$

(سوال ۳)

$$x^2 < |x+4| \xrightarrow{(x) \in \mathbb{R}} x^2 < x+4$$

$$x^2 - x - 4 < 0$$

$$(x-3)(x+2) < 0$$

$$\frac{x}{p} \mid \begin{array}{c} -2 \quad 3 \\ +1 \quad -1 \end{array}$$

$$(-2, 3) \cap \mathbb{I}$$

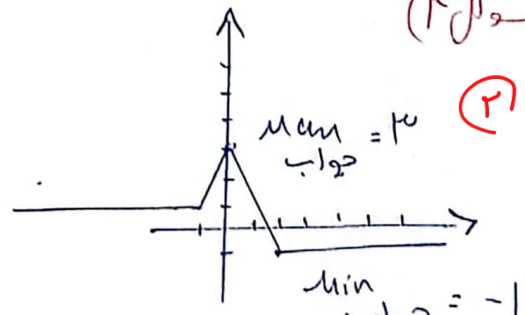
$$\rightarrow x \in (-2, 3)$$

$$|x-a| < \beta \rightarrow -\beta < x-a < \beta$$

$$\alpha - \beta < x < \alpha + \beta \quad \left\{ \begin{array}{l} \alpha + \beta = 3 \\ \alpha - \beta = -2 \end{array} \right\} \rightarrow \alpha = 1$$

$$\alpha = \frac{1}{p} \Rightarrow \beta + \frac{1}{p} = 3 \Rightarrow \beta = \frac{5}{p}$$

$$\begin{cases} x > 2; & (x+1) - (x) + (x-2) = -1 \\ 0 < x < 2; & (x+1) - (x) - (x-2) = -x+3 \\ -1 < x < 0; & (x+1) + (x) - (x-2) = x+3 \\ x < -1; & -(x+1) + (x) - (x-2) = 1 \end{cases}$$

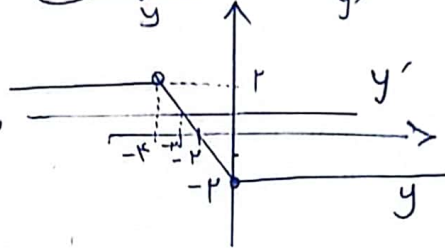


$$\text{مجموع} = 3 + (-1) = 2$$

$$y = \left| \frac{1}{p}x \right| - 2 \xrightarrow{\text{یک طرفه و دیگری منفی}} \left| \frac{1}{p}(x+4) \right| - 2 \xrightarrow{\text{یک طرفه و دیگری منفی}} y = \left| \frac{1}{p}(x+4) \right| - 2 + 1 \quad (2)$$

$$y = y' \Rightarrow \left| \frac{1}{p}x \right| - 2 = \left| \frac{1}{p}x + 2 \right| - 1 \rightarrow \underbrace{\left| \frac{1}{p}x \right| - \left| \frac{1}{p}x + 2 \right|}_{y} = \underbrace{-1}_{y'}$$

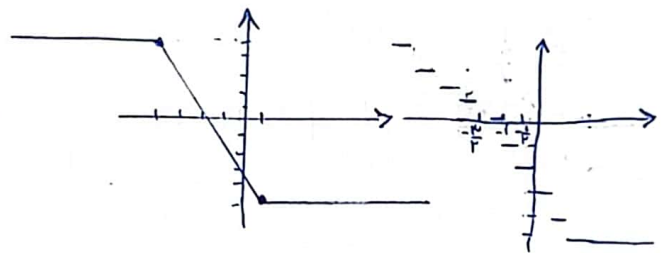
$$y = \begin{cases} x > 0; \left(\frac{1}{p}x\right) - \left(\frac{1}{p}x + 2\right) = -2 \\ -4 < x < 0; -\left(\frac{1}{p}x\right) - \left(\frac{1}{p}x + 2\right) = -x - 2 \\ x < -4; -\left(\frac{1}{p}x\right) + \left(\frac{1}{p}x + 2\right) = 2 \end{cases}$$



$$y' = 1 \quad -x - 2 = 1 \Rightarrow x = -3 \quad \checkmark$$

$$\text{الف) } f(x) = \left[ |1-x| - |x+4| \right]$$

$$\begin{cases} x > 1; (x-1) - (x+4) = -5 \\ -4 < x < 1; -(x-1) - (x+4) = -2x - 3 \\ x < -4; -(x-1) + (x+4) = 5 \end{cases}$$



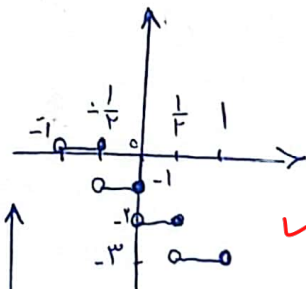
$$= \{-5, -5, -3, -2, -1, 0, 1, 2, 3, 4, 5\} \quad \checkmark$$

$$\text{ب) } f(x) = \frac{1}{n} - \left[ \frac{x+1}{n} \right] = \frac{1}{n} - \left[ 1 + \frac{1}{n} \right] = \frac{1}{n} - \left[ \frac{1}{n} \right] = 0$$

$$0 < \frac{1}{n} - \left[ \frac{1}{n} \right] < 1 \Rightarrow 1 < \frac{1}{n} - \left[ \frac{1}{n} \right] + 1 < 2 \quad \text{Range } [1, 2) \quad \text{سوال 7}$$

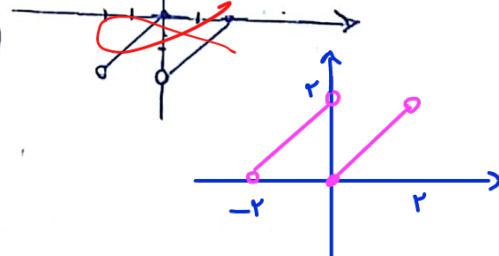
$$\text{الف) } y = \left[ -\frac{x}{2} \right] - 1 \quad x \in (-1, 1)$$

$$\begin{cases} -1 < x < -\frac{1}{2}; 1 - 1 = 0 \\ -\frac{1}{2} < x < 0; 0 - 1 = -1 \\ 0 < x < \frac{1}{2}; -1 - 1 = -2 \\ \frac{1}{2} < x < 1; -2 - 1 = -3 \end{cases}$$



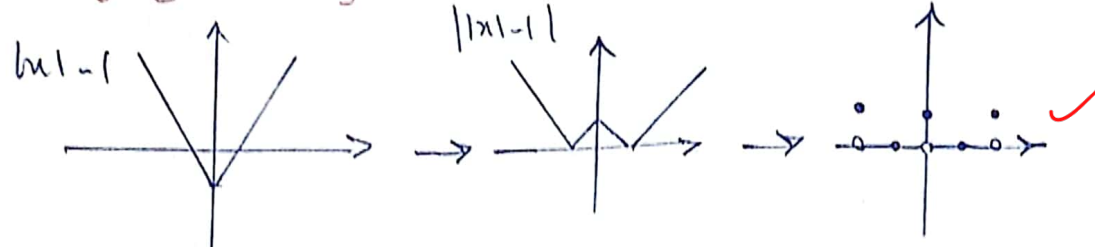
$$\text{ب) } x - 2 \left[ \frac{x}{2} \right] \quad x \in (-2, 2)$$

$$2 \left( \frac{x}{2} \right) - 2 \left[ \frac{x}{2} \right] = 2 \left( \frac{x}{2} - \left[ \frac{x}{2} \right] \right)$$



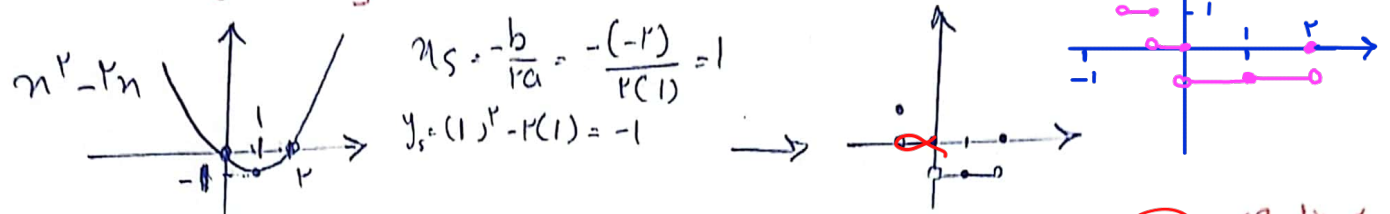
سوال ۸

الف)  $y = [|x| - 1]$   $x \in [-2, 2]$



(۱, ۵)

ب)  $y = [x^2 - 2x]$   $x \in [-1, 2]$



$a + [b] = 4/2 \rightarrow$  قیمت اشیای با ۲۰٪ پستی  
 $b - [a] = 2/4 \rightarrow$  قیمت اشیای با ۵۰٪ پستی

$[a] + [b] = 4$   
 $[b] - [a] = 2 \Rightarrow 2[b] = 6 \rightarrow [b] = 3$   
 $[a] + 3 = 4 \rightarrow [a] = 1$

$a = [a] + 0/2 = 1 + 0/2 = 1/2$   
 $b = [b] + 0/4 = 3 + 0/4 = 3/4$   
 $b - [a] = 2, 4 \rightarrow b = 3, 4$

$\oplus a + b = 1/2 + 3/4 = 7/4$  4, 4

سوال ۱۰

$(1 + \sqrt{2})^6 + (1 - \sqrt{2})^6 = 191$   $[(1 + \sqrt{2})^6] = ?$

$(1 + \sqrt{2})^6 = t \quad t + \frac{1}{t} = 191 \xrightarrow{\times t} t^2 - 191t + 1 = 0 \Rightarrow t = \frac{191 \pm \sqrt{191^2 - 4}}{2}$   
 $\frac{1}{t} = \frac{1}{(1 + \sqrt{2})^6} \times \frac{(1 - \sqrt{2})^6}{(1 - \sqrt{2})^6} = \frac{(\sqrt{2} - 1)^6}{(2 - 1)^6} = (\sqrt{2} - 1)^6$   
 $\Rightarrow t = \frac{191 \pm \sqrt{191^2 - 4}}{2} = 99 \pm \sqrt{191^2 - 4}$   
 $\sqrt{2} \approx 1/41 \Rightarrow 99 + \sqrt{191^2 - 4} = 99 + 191/41 = 99 + 4.66 = 103.66$   
 $[103.66] = 103$