

تکلیف برای سرگیا بکتی / کلاس دوتایی دای

سوال ۱)

$$\frac{1}{n-1} = |n-1| \begin{cases} n > 1 \rightarrow \frac{1}{n-1} = n-1 \xrightarrow{n \neq 1} (n-1)^2 = 1 \\ n^2 - 2n + 1 = 1 \rightarrow n^2 - 2n = 0 \\ n(n-2) = 0 \rightarrow n = 0, 2 \\ \text{حالت دوم} \end{cases}$$

$$\begin{cases} n < 1 \rightarrow \frac{1}{n-1} = -n+1 \xrightarrow{n \neq 1} -(n-1)^2 = 1 \\ n^2 - 2n + 1 = -1 \rightarrow n^2 - 2n + 2 = 0 \\ \Delta = (-2)^2 - 4(1)(2) = -4 < 0 \\ \text{حالت دوم ندارد} \end{cases}$$

در مجموع جواب دارد

سوال ۲)

الف) $\left| \frac{n-1}{n} \right| < 2 \rightarrow \frac{|n-1|}{|n|} < 2 \xrightarrow{|n| > 0} |n-1| < 2|n|$

$$\rightarrow (|n-1|)^2 < (2|n|)^2 \Rightarrow \varepsilon n^2 - \varepsilon n + 1 < 4\varepsilon n^2 \Rightarrow \varepsilon n > 1 \rightarrow n > \frac{1}{\varepsilon}$$

ب) $\left| \frac{n-2}{n+1} \right| > 1 \rightarrow \frac{|n-2|}{|n+1|} > 1 \xrightarrow{|n+1| > 0} |n-2| > |n+1|$

$$\rightarrow (|n-2|)^2 > (|n+1|)^2 \rightarrow n^2 - \varepsilon n + 4 > \varepsilon n^2 + \varepsilon n + 1 \rightarrow n^2 + 1n - 3 < 0$$

$$(n+4)^2 \quad \begin{array}{c|c|c|c} n & -3 & \frac{1}{\varepsilon} & \\ \hline n(n+1) & +1 & -1 & + \end{array}$$

$$(n+9)(n-1) < 0 \rightarrow n = \frac{-9}{\varepsilon} = -3$$

$$n = \frac{1}{\varepsilon}$$

پس: $D_f = (-\infty, -3) \cup (\frac{1}{\varepsilon}, +\infty)$

سوال ۳)

$$n^2 < |n+4| \xrightarrow{(n+4)^2} n^2 < n+4$$

$$n^2 - n - 4 < 0$$

$$(n-3)(n+2) < 0$$

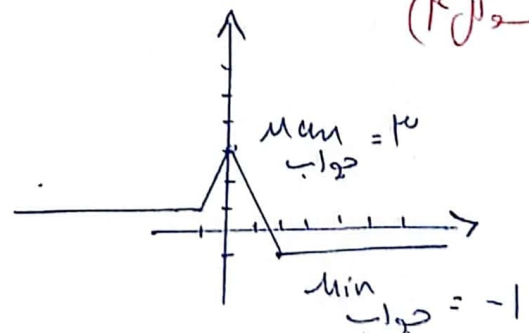
$$\begin{array}{c|c|c|c} n & -2 & 3 & \\ \hline n & +1 & -1 & + \end{array}$$

$$(-2, 3) \cap \mathbb{I} \rightarrow n \in (-2, 3)$$

$$\alpha - \beta < n < \alpha + \beta \quad \begin{cases} \alpha + \beta = 3 \\ \alpha - \beta = -2 \end{cases} \rightarrow \alpha = 1$$

$$\alpha = \frac{1}{p} \Rightarrow \beta + \frac{1}{p} = 3 \Rightarrow \beta = \frac{\omega}{p}$$

$$\begin{cases} n > 2; & (n+1) - (n) + (n-2) = -1 \\ 0 < n < 2; & (n+1) - (n) - (n-2) = -n+2 \\ -1 < n < 0; & (n+1) + (n) - (n-2) = n+3 \\ n < -1; & -(n+1) + (n) - (n-2) = 1 \end{cases}$$

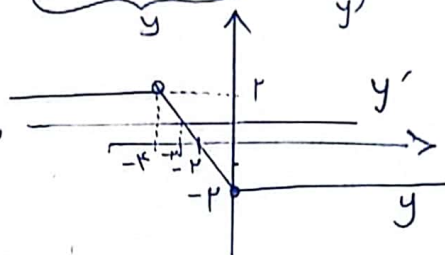


$$\text{مجموع} = 3 + (-1) = 2$$

$$y = \left| \frac{1}{p}x \right| - 2 \xrightarrow{\text{یک واحد به طرف راست}} \left| \frac{1}{p}(x+4) \right| - 2 \xrightarrow{\text{یک واحد به طرف چپ}} y = \left| \frac{1}{p}(x+4) \right| - 2 + 1$$

$$y = y' \Rightarrow \left| \frac{1}{p}x \right| - 2 = \left| \frac{1}{p}x + 2 \right| - 1 \rightarrow \underbrace{\left| \frac{1}{p}x \right| - \left| \frac{1}{p}x + 2 \right|}_{y} = \underbrace{-1}_{y'}$$

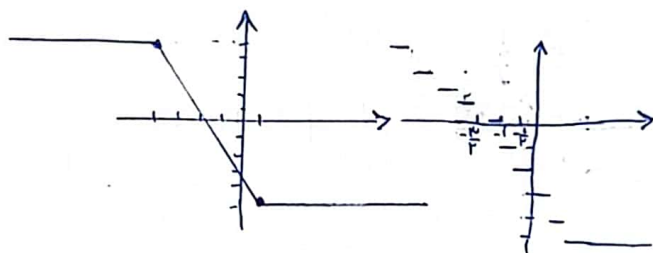
$$y = \begin{cases} x > 0; \left(\frac{1}{p}x\right) - \left(\frac{1}{p}x + 2\right) = -2 \\ -4 < x < 0; -\left(\frac{1}{p}x\right) - \left(\frac{1}{p}x + 2\right) = -x - 2 \\ x < -4; -\left(\frac{1}{p}x\right) + \left(\frac{1}{p}x + 2\right) = 2 \end{cases}$$



$$y' = 1 \quad -x - 2 = 1 \Rightarrow x = -3$$

$$\text{الف) } f(x) = \left[|1-x| - |x+4| \right]$$

$$\begin{cases} x > 1; (x-1) - (x+4) = -5 \\ -4 < x < 1; -(x-1) - (x+4) = -2x - 3 \\ x < -4; -(x-1) + (x+4) = 5 \end{cases}$$



$$= \{-5, -5, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

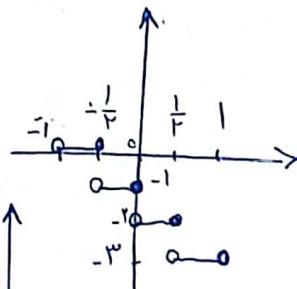
$$\text{ب) } f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] = \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left[\frac{1}{x} \right] + 1$$

$$0 < \frac{1}{x} - \left[\frac{1}{x} \right] < 1 \Rightarrow 1 < \frac{1}{x} - \left[\frac{1}{x} \right] + 1 < 2 \quad \text{Range } [1, 2)$$

(سوال 7)

$$\text{الف) } y = \left[-\frac{1}{x} \right] - 1 \quad x \in (-1, 1)$$

$$\begin{cases} -1 < x < -\frac{1}{2}; 1 - 1 = 0 \\ -\frac{1}{2} < x < -\frac{1}{3}; 0 - 1 = -1 \\ 0 < x < \frac{1}{2}; -1 - 1 = -2 \\ \frac{1}{2} < x < 1; -2 - 1 = -3 \end{cases}$$



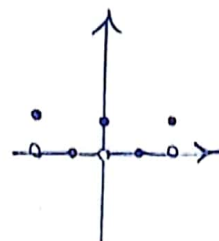
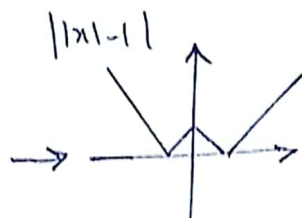
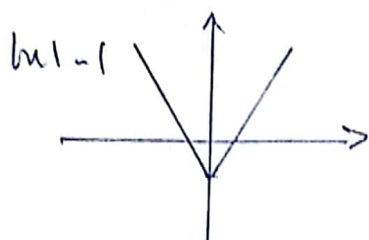
$$\text{ب) } x - 2 \left[\frac{x}{2} \right] \quad x \in (-2, 2)$$

$$2 \left(\frac{x}{2} \right) - 2 \left[\frac{x}{2} \right] = 2 \left(\frac{x}{2} - \left[\frac{x}{2} \right] \right)$$

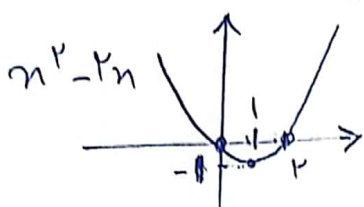


سوال ۸

$$y = [|x| - 1] \quad x \in [-2, 2]$$

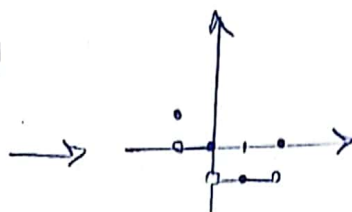


$$y = [x^2 - 2x] \quad x \in [-1, 2]$$



$$x_s = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

$$y_s = (1)^2 - 2(1) = -1$$



$$a + [b] = 4/2$$

$$b - [a] = 2/4$$

قیمت اعظمی
با ۰/۲ است

قیمت اعظمی
با ۰/۴ است

$$[a] + [b] = 2$$

$$[b] - [a] = 1$$

$$\begin{cases} [a] + [b] = 2 \\ [b] - [a] = 1 \end{cases} \Rightarrow \begin{aligned} 2[b] &= 3 \rightarrow [b] = 1.5 \\ [a] + 1.5 &= 2 \rightarrow [a] = 0.5 \end{aligned}$$

$$a = [a] + 0/2 = 1 + 0/2 = 1/2$$

$$b = [b] + 0/4 = 1.5 + 0/4 = 1.5$$

$$\oplus \quad a + b = 1/2 + 1.5 = 1.75 = \frac{7}{4}$$

$$(1 + \sqrt{2})^6 + (1 - \sqrt{2})^6 = 198$$

$$[(1 + \sqrt{2})^6] = ?$$

سوال ۱۰

$$(1 + \sqrt{2})^6 = t \quad t + \frac{1}{t} = 198 \Rightarrow t^2 - 198t + 1 = 0 \Rightarrow t = \frac{198 \pm \sqrt{198^2 - 4}}{2}$$

$$\frac{1}{t} = \frac{1}{(1 + \sqrt{2})^6} \times \frac{(1 - \sqrt{2})^6}{(1 - \sqrt{2})^6} = \frac{(1 - \sqrt{2})^6}{(1 + \sqrt{2})^6} = (1 - \sqrt{2})^6$$

$$\begin{aligned} \Rightarrow t &= \frac{198 \pm \sqrt{198^2 - 4}}{2} \\ &= \frac{198 \pm 198\sqrt{2}}{2} = 99 \pm 99\sqrt{2} \\ \sqrt{2} &\approx 1/1.414 \Rightarrow 99 + 99 \times 1.414 = 99 + 139.986 = 139.986 \end{aligned}$$

$$[139.986] = 139$$