

$AE = x$
 $BE = \sqrt{9x^2 + x^2} = x\sqrt{10}$
 $\triangle BFC \sim \triangle AEF \Rightarrow \frac{BF}{FE} = \frac{3x}{x} = 3 \Rightarrow BF = 3FE$
 $BF + FE = x\sqrt{10} \Rightarrow FE = \frac{x\sqrt{10}}{4}$
 $AC = \sqrt{9x^2 + 9x^2} = 3\sqrt{2}x$
 $\triangle BFC \sim \triangle AEF \Rightarrow \frac{CF}{AF} = \frac{3x}{x} = 3 \Rightarrow CF = 3AF$
 $CF + FA = 3\sqrt{2}x \Rightarrow AF = \frac{3\sqrt{2}}{4}x$

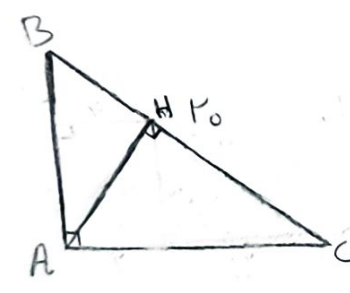
$\frac{EF}{AF} = \frac{\frac{\sqrt{10}}{4}x}{\frac{3\sqrt{2}}{4}x} = \frac{\sqrt{10}}{3\sqrt{2}} = \frac{\sqrt{5}}{3}$ ✓

$\triangle AED \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{y}{2x} = \frac{x}{3\sqrt{2}x} \Rightarrow y = \frac{2}{3\sqrt{2}}x$

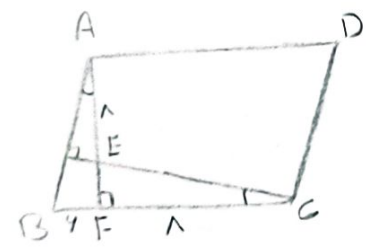
$DE \parallel BC \Rightarrow \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow \frac{x}{3\sqrt{2}x} = \frac{DE}{3x} \Rightarrow DE = \frac{1}{\sqrt{2}}x$

$\triangle HDE \sim \triangle HFC \Rightarrow \frac{DE}{FC} = \frac{HE}{HF} = \frac{HD}{HC}$

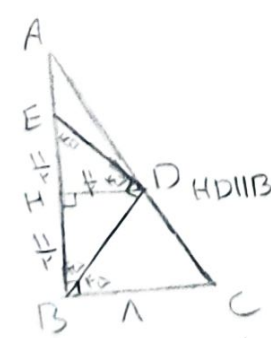
$BC = 3DE$
 $BF + FC = 3DE$
 $3 + FC = \frac{9}{\sqrt{2}}FC$
 $3 = \frac{8}{\sqrt{2}}FC \Rightarrow FC = \frac{3\sqrt{2}}{8}$
 $\Rightarrow BC = 3 + \frac{3\sqrt{2}}{8} = 4.106$ ✓



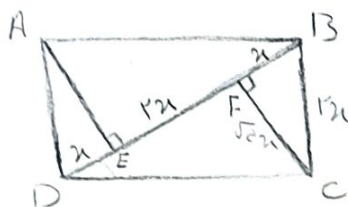
$(AB)^2 = BH \times BC = 1$
 $(AC)^2 = CH \times BC = 3$
 $\frac{AB}{AC} = \sqrt{\frac{1}{3}}$



$\triangle EFC \sim \triangle AFB \Rightarrow \frac{FC}{AF} = \frac{EF}{BF} \Rightarrow \frac{3}{1+EF} = \frac{EF}{4}$
 $EF^2 + 4EF - 12 = 0$
 $(EF+6)(EF-2) = 0 \Rightarrow EF = 2$ ✓
 $AF = 1+2 = 3$ ✓



$\triangle AHD \sim \triangle BHC \Rightarrow \frac{AH}{HB} = \frac{HD}{HC} \Rightarrow \frac{AE + \frac{11}{3}}{AE + 11} = \frac{11}{3}$
 $11AE + 121 = \frac{11}{3}AE + \frac{121}{3} \Rightarrow \frac{11}{3}AE = \frac{121}{3} \Rightarrow AE = 11$ ✓



$$DE = x$$

$$FC^2 = FD \cdot FB \Rightarrow FC = \sqrt{3}x$$

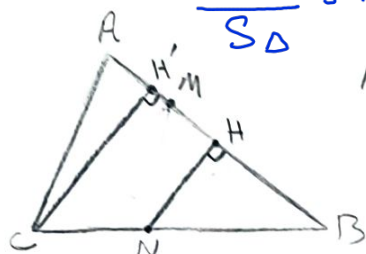
$$BC^2 = BF^2 + CF^2 \Rightarrow BC = 2x$$

$$DC^2 = DF^2 + CF^2 \Rightarrow DC = 2\sqrt{3}x$$

نسبت رو برعکس نوشت!

$$\frac{S_{\square}}{S_{\Delta}} = 1$$

$$\frac{S_{\triangle BCF}}{S_{\triangle ABCD}} = \frac{\frac{1}{2} \cdot 2x \cdot \sqrt{3}x}{2x \cdot 2\sqrt{3}x} = \frac{1}{4} \quad \checkmark \quad \text{ارفاق}$$

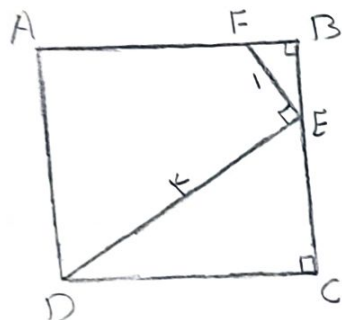


$$\triangle NHB \sim \triangle CH'B \Rightarrow \frac{NH}{CH'} = \frac{BN}{BC} = \frac{BN}{BN+CN} = \frac{BN}{BN+\frac{1}{2}BN} = \frac{2}{3}$$

$$\frac{S_{\triangle MNB}}{S_{\triangle ABC}} = \frac{NH}{CH'} \times \frac{MB}{AB} = \frac{2}{3} \times \frac{1/3}{1} = \frac{1}{3}$$

$$\Rightarrow \frac{MB}{AB} = \frac{1}{9} \quad \frac{MB}{AB-MB} = \frac{1}{9-1}$$

$$\Rightarrow \frac{MB}{AM} = \frac{1}{8} \quad \checkmark$$



$$\angle FEB + \angle FED + \angle DEC = 180^\circ$$

$$\left. \begin{aligned} \angle FEB + \angle DEC &= 90^\circ \\ \angle FEB + \angle FED &= 90^\circ \end{aligned} \right\} \Rightarrow \angle DEC = \angle FED \quad \text{و} \quad \angle B = \angle C = 90^\circ \Rightarrow \triangle FEB \sim \triangle DEC$$

$$\Rightarrow \frac{FB}{EC} = \frac{BE}{CD} = \frac{1}{k} \Rightarrow \begin{cases} FB = EC \\ BE = CD \\ BE = BE + EC \end{cases} \Rightarrow FB = 3BE$$

$$FB^2 + BE^2 = 1$$

$$\left(\frac{3}{k}BE\right)^2 + BE^2 = 1$$

$$\frac{9}{k^2}BE^2 + BE^2 = 1 \Rightarrow BE = \frac{k}{10} \Rightarrow EC = 3 \times \frac{k}{10} = \frac{3k}{10} \quad S_{\triangle ABCD} = BC^2 = (1 + 2 + k)^2 = 10, 2k \quad \checkmark$$

۱- در مثلث AGC به علت اینکه GE=EC است AE میانه است و چون BD میانه است پس AD=DC و GD میانه دیگری در مثلث AGC است مثل برخورد دو میانه است پس داریم! (۲)

$$\frac{FD}{DG} = \frac{1}{3}$$

$$\frac{DG}{BD} = \frac{1}{3} \Rightarrow \frac{FD}{DG} \times \frac{DG}{BD} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} = \frac{FD}{BD} \Rightarrow \frac{BD}{FD} = 9 \quad \checkmark$$

BD میانه است پس داریم!