محمدحسین ایروانی

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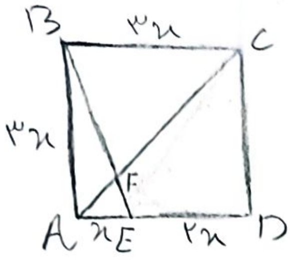

$AE = x$

$BE = \sqrt{9x^2 + x^2} = x\sqrt{10}$

$\hat{BFC} \sim \hat{AEF} \Rightarrow \frac{BF}{FE} = \frac{3x}{x} = 3 \Big\} \Rightarrow \begin{matrix} BF + FE = x\sqrt{10} \\ BF = 3FE \end{matrix} \Big\} \Rightarrow FE = \frac{x\sqrt{10}}{4}$

$AC = \sqrt{9x^2 + 9x^2} = 3\sqrt{2}\, x$

$\hat{BFC} \sim \hat{AEF} \Rightarrow \frac{CF}{AF} = \frac{3x}{x} = 3 \Big\} \Rightarrow \begin{matrix} CF + FA = 3\sqrt{2}\, x \\ CF = 3AF \end{matrix} \Big\} \Rightarrow AF = \frac{3\sqrt{2}}{4} x$

$\frac{EF}{AF} = \frac{\frac{\sqrt{10}}{\cancel{4}}\cancel{x}}{\frac{3\sqrt{2}}{\cancel{4}}\cancel{x}} = \frac{\sqrt{5}}{3}$

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$\hat{AED} \sim \hat{ABC} \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{2}{x+1} = \frac{x}{15}$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$\boxed{x = 5}$

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$DE \parallel BC \Rightarrow \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow \frac{x}{x+1.2y} = \frac{x}{x + 1.2 \times \frac{5}{3}x} = \frac{x}{3x} = \frac{1}{3} = \frac{DE}{BC}$

$\hat{HDE} \sim \hat{HFC} \Rightarrow \frac{DE}{FC} = \frac{x}{y} = \frac{x}{\frac{5}{3}x} = \frac{3}{5}$

$BC = 3DE$

$BF + FC = 3DE$

$3 + FC = \frac{9}{5} FC$

$3 = \frac{4}{5} FC \quad FC = \frac{15}{4}$

$\Rightarrow BC = 3 + \frac{15}{4} = 6.75$

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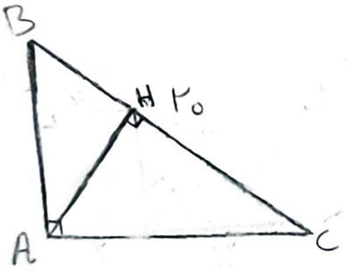

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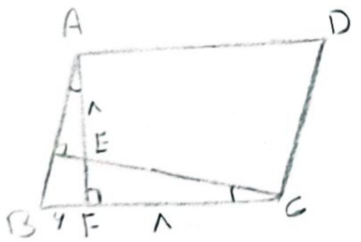

$\hat{EFC} \sim \hat{ABF} \Rightarrow \frac{FC}{AF} = \frac{EF}{BF} \Rightarrow \frac{8}{8+EF} = \frac{EF}{6}$

$EF^2 + 8EF - 48 = 0$

$(EF + 12)(EF - 4) = 0$

$EF = 4$

$AF = 8 + 4 = 12$

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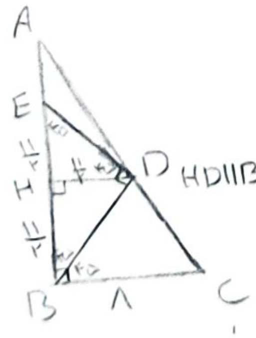

$HD \parallel BC \Rightarrow \frac{AH}{AB} = \frac{HD}{BC} \Rightarrow \frac{AE + \frac{11}{2}}{AE + 11} = \frac{\frac{11}{2}}{8}$

$8AE + 44 = \frac{11}{2}AE + \frac{121}{2}$

$\frac{5}{2}AE = \frac{33}{2} \quad AE = \frac{33}{5} = 6.6$

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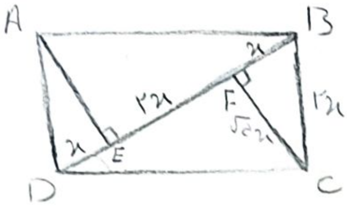

$DE = x$

$FC^2 = FD \cdot FB \Rightarrow FC = \sqrt{3}\,x$

$BC^2 = BF^2 + CF^2 \Rightarrow BC = 2x$

$DC^2 = DF^2 + CF^2 \Rightarrow DC = 2\sqrt{3}\,x$

$$\frac{S_{BCF}}{S_{ABCD}} = \frac{\frac{\sqrt{3}x^2}{2}}{4\sqrt{3}\,x^2} = \frac{1}{8}$$

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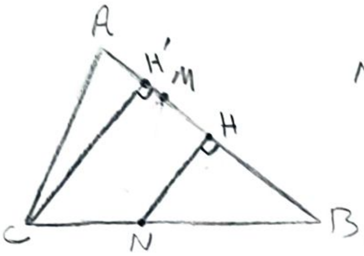

$$\overset{\triangle}{NHB} \sim \overset{\triangle}{CH'B} \Rightarrow \frac{NH}{CH'} = \frac{BN}{BC} = \frac{BN}{BN+CN} = \frac{BN}{BN+\frac{2}{3}BN} = \frac{3}{5}$$

$$\frac{S_{MNB}}{S_{ABC}} = \frac{NH}{CH'} \times \frac{MB}{AB} = \frac{3}{5} \times \frac{MB}{AB} = \frac{1}{3}$$

$$\Rightarrow \frac{MB}{AB} = \frac{5}{9} \qquad \frac{MB}{AB-MB} = \frac{5}{9-5}$$

$$\Rightarrow \frac{MB}{AM} = \frac{5}{4}$$

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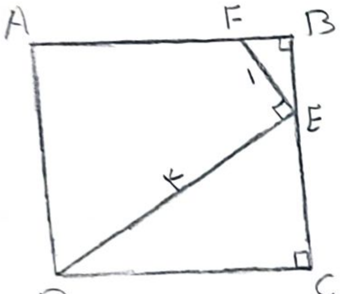

$$F\hat{E}B + \underset{90}{\cancel{F\hat{E}D}} + D\hat{E}C = 180$$

$$\left.\begin{array}{l} F\hat{E}B + D\hat{E}C = 90 \\ F\hat{E}B + E\hat{F}B = 90 \end{array}\right\} \Rightarrow \left.\begin{array}{l} D\hat{E}C = E\hat{F}B \\ \hat{B} = \hat{C} = 90 \end{array}\right\} \xRightarrow{\text{زز}} \overset{\triangle}{FBE} \sim \overset{\triangle}{DEC}$$

$$\Rightarrow \frac{FB}{EC} = \frac{BE}{CD} = \frac{1}{4} \Rightarrow \left.\begin{array}{l} 4FB = EC \\ 4BE = CD \\ 4BE = BE + EC \\ 3BE = EC \end{array}\right\} \Rightarrow 4FB = 3BE$$

$FB^2 + BE^2 = 1$

$(\frac{3}{4}BE)^2 + BE^2 = 1$

$\frac{25}{16}BE^2 = 1 \Rightarrow BE = \frac{4}{5} = 0.8 \Rightarrow EC = 3 \times \frac{4}{5} = \frac{12}{5} = 2.4 \quad S_{ABCD} = BC^2 = (0.8 + 2.4)^2 = 10.24$

۱۰- در مثلث AGC به علت اینکه $GE=EC$ است AE میانه است و چون BD میانه است پس $AD=DC$
و GD میانه دیگری در مثلث AGC است محل برخورد دو میانه F است پس داریم

$$\frac{FD}{DG} = \frac{1}{3}$$

BD میانه است پس داریم

$$\frac{DG}{BD} = \frac{1}{3} \Rightarrow \frac{FD}{DG} \times \frac{DG}{BD} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} = \frac{FD}{BD} \Rightarrow \frac{BD}{FD} = 9$$