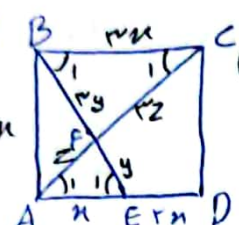
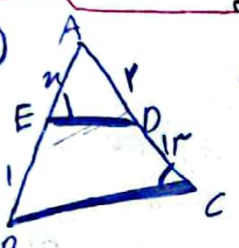
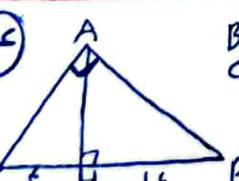
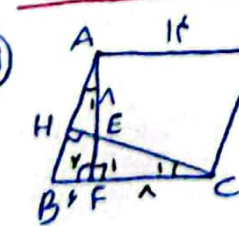


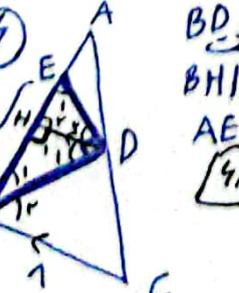
①  $AE + ED = AD = 3n = AB$
 $AE \parallel BC, BE \Rightarrow \hat{B}_1 = \hat{E}_1$
 $AE \parallel BC, AC \Rightarrow \hat{C}_1 = \hat{A}_1$
 $\Rightarrow AEF \sim CBF \Rightarrow \frac{BF}{AE} = \frac{BC}{FA} = \frac{FC}{FA} = 3$
 $\Rightarrow BF = 3EF = 3y$ و $FC = 3FA = 3z$
 $ABE: AB^2 + AE^2 = BE^2 \Rightarrow 9n^2 + n^2 = 10y^2 \Rightarrow 10n^2 = 10y^2 \Rightarrow EF = y = \frac{\sqrt{10}}{2}n$
 $ABC: AB^2 + BC^2 = AC^2 \Rightarrow 9n^2 + n^2 = 10z^2 \Rightarrow 10n^2 = 10z^2 \Rightarrow AF = z = \frac{\sqrt{10}}{2}n = \frac{3\sqrt{10}}{2}n$
 $\Rightarrow \frac{EF}{AF} = \frac{\frac{\sqrt{10}}{2}n}{\frac{3\sqrt{10}}{2}n} = \frac{\sqrt{10}}{3}$ ✓

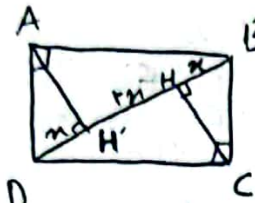
②  $A\hat{E}D = A\hat{C}B$
 $\hat{A} = \hat{A}$
 $A\hat{E}D = A\hat{C}B$
 $\Rightarrow AED \sim ACB \Rightarrow \frac{AE}{AC} = \frac{AD}{AB}$
 $\Rightarrow (n)(n+1) = 20 = 4 \times 5 \Rightarrow n = 4$ ✓

③  $BF = 3n$
 $DE \parallel BC$
 $BC = ?$
 $DE \parallel BC \Rightarrow \frac{AE}{AC} = \frac{DE}{BC} = \frac{n}{3n} = \frac{1}{3}$
 $\Rightarrow DE = \frac{1}{3}BC = n$ (I)
 $DE \parallel BC, EF \Rightarrow \hat{E}_1 = \hat{F}_1$
 $DE \parallel BC, DC \Rightarrow \hat{D}_1 = \hat{C}_1$
 $\Rightarrow DHE \sim CHF \Rightarrow \frac{EH}{FC} = \frac{DE}{FC} = \frac{n}{3n} = \frac{1}{3}$
 $\Rightarrow DE = \frac{1}{3}FC = n$ (II)
 $BC = BF + FC \Rightarrow 3n = \frac{1}{3}k + 3 \Rightarrow \frac{1}{3}k = 3 \Rightarrow k = 9 \Rightarrow 3k = \frac{27}{3} = 9 = BC$ ✓

④  $BH = 15$
 $CH = 4$
 $\frac{AB}{AC} = ?$
 $AB^2 = BH \times BC$
 $AC = CH \times BC$
 $\Rightarrow \left(\frac{AB}{AC} \right)^2 = \frac{BH}{CH} = \frac{15}{4}$
 $\frac{AB}{AC} = \frac{3}{2}$

⑤  $AD = 12$
 $AE = 4$
 $BF = 3$
 $AF = ?$
 $ABCD \Rightarrow AD = BC = 12$
 $BC = BF + FC = 3 + FC \Rightarrow FC = 9$
 $B\hat{H}C: \hat{B} + \hat{C} = 90^\circ \Rightarrow \hat{A}_1 = \hat{C}_1$
 $A\hat{F}B: \hat{B} + \hat{A}_1 = 90^\circ \Rightarrow \hat{F}_1 = \hat{F}_1$
 $\Rightarrow ABF \sim CFE$
 $\frac{BF}{FE} = \frac{AF}{FC}$
 $\Rightarrow (n)(n+1) = 12 = 3 \times 4 \Rightarrow n = 3 = FE$ (I)
 $AF = FE + AE = 3 + 4 = 12$ ✓

⑥  $BD \perp AC$
 $BH \parallel BC$
 $AE = ?$
 $BD \perp AC \Rightarrow \hat{B}_1 = \hat{B}_1$
 $BH \parallel BC, BD \Rightarrow \hat{B}_1 = \hat{D}_1$
 $\Rightarrow \hat{D}_1 = 45^\circ$
 $\hat{A}_1 = 90^\circ, \hat{E}_1 = 180^\circ - \hat{D}_1 - \hat{A}_1 = 45^\circ$
 $\Rightarrow \hat{E}_1 = 45^\circ$
 $\Rightarrow BDE$ is an isosceles triangle
 $\Rightarrow DH = BH = 4 = HD$
 $HD \parallel BC \Rightarrow \frac{HA}{AC} = \frac{HD}{BC} = \frac{4}{12} = \frac{1}{3}$
 $\Rightarrow \frac{AE}{AC} = \frac{1}{3} \Rightarrow AE = 4$ ✓

(V) 

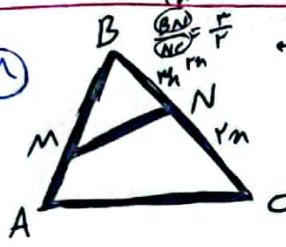
$S_{ABD} = \frac{AD \times AB}{2}$
 $S_{BCD} = \frac{BC \times CD}{2}$
 $ABCD \Rightarrow BC=AD, DC=AB$

$\Rightarrow S_{ABD} = S_{BCD}$
 $S_{ABCD} = S_{ABD} + S_{BCD} \Rightarrow S_{ABCD} = 2S_{ABD}$ (2)

$S_{ABD} = \frac{AH \times BD}{2}$
 $S_{ADH'} = \frac{AH' \times DH'}{2}$

$\Rightarrow \frac{S_{ABD}}{S_{ADH'}} = \frac{BD}{DH'} = \frac{r}{n} = r \Rightarrow \frac{S_{ABCD}}{S_{ADH'}} = \frac{2S_{ABD}}{S_{ADH'}} = 2 \times r = 2$ (1) ✓

نسبت مساحت مستطیل به مساحت لوزی برابر شد

(1) 

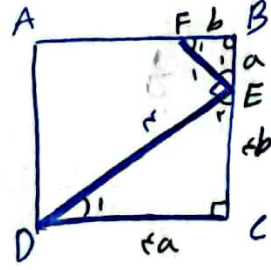
$\angle BNM = \angle NC$
 $S_{ABC} = 3S_{BMN}$
 $\frac{BM}{AM} = ?$

$S_{ABC} = \frac{1}{2} AB \times BC \sin B$
 $S_{BMN} = \frac{1}{2} BM \times BN \sin B$

$\Rightarrow \frac{S_{ABC}}{S_{BMN}} = \frac{AB \times BC}{BM \times BN} = 3$ (2)

$\Rightarrow \frac{AB}{BM} \times \frac{BC}{BN} = 3 \Rightarrow \frac{AB}{BM} = \frac{3}{1} \Rightarrow \frac{AB}{BM} = 3$

$\Rightarrow \frac{AM}{BM} = \frac{1}{2} \Rightarrow \frac{BM}{AM} = \frac{2}{1}$ ✓

(9) 

$S_{ABCD} = ?$
 $\angle BEF = \angle E_1 + \angle F_1 = 90^\circ$
 $\angle E_1 + \angle E_2 + 90^\circ = 180^\circ \Rightarrow \angle E_1 + \angle E_2 = 90^\circ$

$\Rightarrow \angle F_1 = \angle E_2$
 $\angle B = \angle C = 90^\circ$

$\Rightarrow \triangle BEF \sim \triangle CED$ (2)

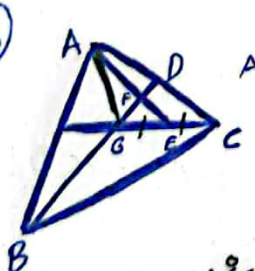
$\frac{BF}{CE} = \frac{FE}{ED} = \frac{BE}{CD} = \frac{1}{r}$

$\Rightarrow \frac{BF}{a} = \frac{1}{r} \Rightarrow BF = \frac{a}{r}$
 $\frac{CE}{b} = \frac{1}{r} \Rightarrow CE = \frac{b}{r}$

$BE^2 + BF^2 = EF^2 \Rightarrow a^2 + \frac{a^2}{r^2} = 1$

$BC = DC \Rightarrow a = a + \frac{b}{r} \Rightarrow b = \frac{r}{2}a$

$\Rightarrow DC = a = \frac{1}{2} \Rightarrow S_{DC} = \frac{r \times \frac{1}{2}}{2} = \frac{r}{4}$ ✓

(10) 

$GE = EC$
 $\angle G = \angle C$
 $\frac{BD}{FD} = ?$

$\triangle AGC: GE = EC \Rightarrow AE$ میانه دارد بر ضلع AC است GC

$\triangle AGC: \angle G = \angle C \Rightarrow BD$ میانه دارد بر ضلع AC است AC (در $\triangle ABC$)

$\Rightarrow AD = DC \Rightarrow GD$ میانه دارد بر ضلع AC است $\triangle AGC$ در ضلع AC

$\Rightarrow \triangle AGC$ مرکز ثقل F است

$\frac{GF}{FD} = \frac{1}{2} \Rightarrow \frac{GF+FD}{FD} = \frac{1+1}{1} \Rightarrow \frac{GD}{FD} = 2$

$\triangle ABC$ مرکز ثقل G است

$\frac{BG}{GD} = \frac{2}{1} \Rightarrow \frac{BG+GD}{GD} = \frac{2+1}{1} \Rightarrow \frac{BD}{GD} = 3$

$\Rightarrow \frac{GF}{FD} \times \frac{BD}{GF} = \frac{BD}{FD} = 2 \times 3 = 6$ ✓