

$$\begin{aligned} BF &= r \\ r y &= d n \quad n = r a \implies y = d a \\ DE &\parallel BC \\ BC &=? \end{aligned}$$

$DE \parallel BC, \text{ موسط } EF \Rightarrow \hat{E}_1 = \hat{F}_1 \} \Rightarrow DHE \sim CHF \Rightarrow \frac{EH}{FH} = \frac{DE}{FC} = \frac{n}{y} = \frac{r}{s}$
 $DE \parallel BC, \text{ موسط } DC \Rightarrow \hat{D}_1 = \hat{C}_1 \} \Rightarrow \frac{DE}{FC} = \frac{EH}{FH} = \frac{n}{y} = \frac{r}{s}$
 $\Rightarrow \frac{DE}{K} = \frac{FC}{\frac{1}{K}} = K \text{ (II)}$

(12)

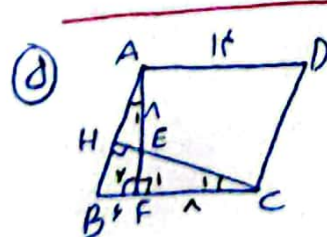


$BH = 15$
 $CH = 4$
 $\frac{AB}{AC} = ?$

حلقه در رابطه طولی در مثلث قائم الزاویه داریم:

$AB = BH \times BC$
 $AC = CH \times BC$

$\Rightarrow \frac{AB}{AC} = \frac{BH}{CH} = 4$



$AD=12$
 $AE=8$
 $BF=5$
 $AF=?$

$\left\{ \begin{array}{l} \text{ABCD متوازي الاضلاع} \\ \Rightarrow AD=BC=12 \\ BC=BF+FC=5+FC \end{array} \right\} \Rightarrow FC=7$

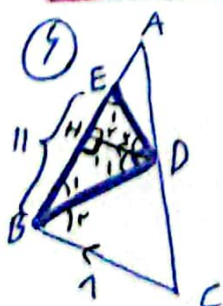
$\left\{ \begin{array}{l} \angle BHC: \hat{B} + \hat{C} = 90^\circ \\ \angle AFB: \hat{B} + \hat{A} = 90^\circ \end{array} \right\} \Rightarrow \hat{A}_1 = \hat{C}_1 \Rightarrow \triangle ABF \sim \triangle CFE$

$\hat{F}_1 = \hat{F}_2 = 90^\circ$

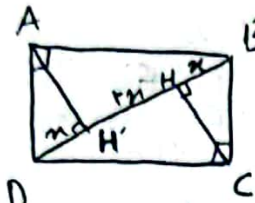
$\frac{BF}{FE} = \frac{AF}{FC}$

$5 = x \times 12 \Rightarrow \sqrt{12^2 - 5^2} = FE$

$$AF = FE + AE \quad \text{--- (I)} \quad 6 + 7 = 13$$



$BD \parallel AC$
 $BH \parallel BC$
 $AE = ?$
 $\angle 4, 6$
 $BD \parallel AC \Rightarrow \hat{B}_1 = \hat{B}_r$
 $BH \parallel BC$ و BD مبرب $\Rightarrow \hat{B}_r = \hat{D}_1$
 $\Rightarrow \hat{D}_r = 45^\circ$
 $A_r = 90^\circ, \hat{E}_1 = 180^\circ - \hat{D}_r + A_r$
 $\Rightarrow DH = BH = 1$
 $HD \parallel BC$
 $\frac{HA}{BA} = \frac{HD}{BC} = \frac{1}{2}$
 $AE = 1, 5$

⑤ 

$$S_{ABD} = \frac{AD \times AB}{2}$$

$$S_{BCD} = \frac{BC \times CD}{2}$$

$$ABCD \Rightarrow BC=AD, DC=AB$$

$$\Rightarrow S_{ABD} = S_{BCD}$$

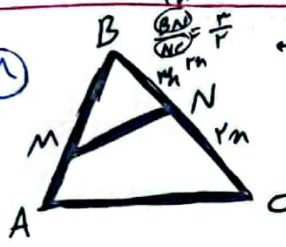
$$S_{ABCD} = S_{ABD} + S_{BCD} \Rightarrow S_{ABCD} = 2S_{ABD}$$

$$S_{ABD} = \frac{AH \times BD}{2}$$

$$S_{ADH'} = \frac{AH' \times DH'}{2}$$

$$\Rightarrow \frac{S_{ABD}}{S_{ADH'}} = \frac{BD}{DH'} = \frac{2n}{n} = 2 \Rightarrow \frac{S_{ABCD}}{S_{ADH'}} = \frac{2S_{ABD}}{S_{ADH'}} = 2 \times 2 = 4$$

نسبت مساحت مستطیل به مساحت کوچکترین مثلث

⑥ 

$$2BN = 3NC$$

$$S_{ABC} = 3S_{BMN}$$

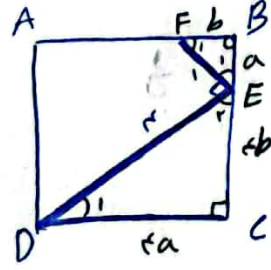
$$\frac{BM}{AM} = ?$$

$$S_{ABC} = \frac{1}{2} AB \times BC \sin B$$

$$S_{BMN} = \frac{1}{2} BM \times BN \sin B$$

$$\Rightarrow \frac{S_{ABC}}{S_{BMN}} = \frac{AB}{BM} \times \frac{BC}{BN} = 3$$

$$\Rightarrow \frac{AB}{BM} \times \frac{3}{2} = 3 \Rightarrow \frac{AB}{BM} = 2 \Rightarrow \frac{BM}{AM} = \frac{1}{1}$$

⑦ 

$$S_{ABCD} = ?$$

$$\widehat{BFE} : \widehat{E_1} + \widehat{F_1} = 90^\circ$$

$$\widehat{E_1} + \widehat{E_2} + 90^\circ = 180^\circ \Rightarrow \widehat{E_1} + \widehat{E_2} = 90^\circ$$

$$\widehat{F_1} = \widehat{E_2}$$

$$\widehat{B} = 2 \times 90^\circ$$

$$\Rightarrow BFE \sim CED$$

$$\frac{BF}{CE} = \frac{FE}{ED} = \frac{BE}{CD} = \frac{1}{2}$$

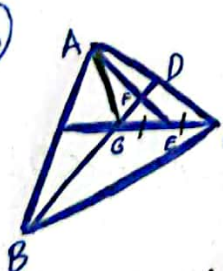
$$\Rightarrow BE = CD = 2a, BF = CE = 2b$$

$$BE^2 = BF^2 + FE^2 \Rightarrow a^2 + b^2 = 1$$

$$\Rightarrow a^2 + \frac{1}{4}a^2 = 1 \Rightarrow \frac{5}{4}a^2 = 1 \Rightarrow a^2 = \frac{4}{5} \Rightarrow a = \frac{2}{\sqrt{5}}$$

$$BC = DC \Rightarrow 2a = a + 2b \Rightarrow b = \frac{1}{2}a$$

$$\Rightarrow DC = 2a = \frac{4}{\sqrt{5}} \Rightarrow S_{ABCD} = \frac{4 \times 4}{5} = 10.24$$

⑧ 

$$GE = EC$$

$$A\widehat{B}C$$
 مرکز ثقل G
$$\frac{BD}{FD} = ?$$

$$A\widehat{G}C : GE = EC \Rightarrow AE$$
 میانه دارد بر ضلع و در آن است GC
$$A\widehat{B}C \Rightarrow BD$$
 میانه دارد بر ضلع و در آن است AC (در A\widehat{B}C)
$$\Rightarrow AD = DC \Rightarrow GD$$
 میانه دارد بر ضلع AC در ضلع A\widehat{B}C است
$$\Rightarrow A\widehat{B}C$$
 مرکز ثقل F است
$$\Rightarrow \frac{GF}{FD} = \frac{2}{1} \Rightarrow \frac{GF+FD}{FD} = \frac{2+1}{1} \Rightarrow \frac{GD}{FD} = 3$$

$$A\widehat{B}C$$
 مرکز ثقل G است
$$\Rightarrow \frac{BG}{GD} = \frac{2}{1} \Rightarrow \frac{BG+GD}{GD} = \frac{2+1}{1} \Rightarrow \frac{BD}{GD} = 3$$

$$\Rightarrow \frac{GF}{FD} \times \frac{BD}{GF} = \frac{BD}{FD} = 3 \times 3 = 9$$