

$\hat{A}FE = \hat{B}FC, \hat{B}CF = \hat{F}AE$ (قطریم) $\Rightarrow \hat{A}FE \cong \hat{B}FC$ $\left\{ \begin{array}{l} x = AE \\ \text{شکل مربع باشد} \end{array} \right\}$ فرض ۱

$BC = AE + ED \rightarrow BC = 3AE \rightarrow \frac{BC}{AE} = 3$ نسبت مشابه

$AC^2 = AD^2 + CD^2 \xrightarrow{AC=5} 3\sqrt{2}x \rightarrow 3\sqrt{2}x = AF = CF, \frac{CF}{AF} = \frac{1}{1} \rightarrow AF = \frac{3\sqrt{2}x}{2}$

$BE^2 = EA^2 + BA^2 \rightarrow BE = x\sqrt{2} = BE + FE, \frac{BE}{FE} = \frac{1}{1} \rightarrow FE = \frac{x\sqrt{2}}{2}$

$EF = AF = \frac{3\sqrt{2}x}{2} \times \frac{1}{2} = \frac{3\sqrt{2}x}{4} \rightarrow \frac{50}{C} \checkmark$

$E = C, \hat{A} = \hat{A} \rightarrow \hat{A}ED \cong \hat{ACB} \rightarrow \frac{2}{x+1} = \frac{x}{10} \rightarrow x^2 + x - 20 = 0$

$(x+6)(x-4) = 0 \rightarrow x = 4$ (چون $x > 0$) \checkmark

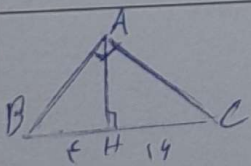
$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\frac{x}{x+1} = \frac{DE}{FC+C}, \frac{DE}{FC} = \frac{x}{8} \rightarrow \frac{FC+C}{DE} = \frac{x+1}{8} = \frac{2x}{8} \rightarrow 3DE = FC+C$

$5DE = 3FC \rightarrow 9DE - 9 = 5DE \rightarrow DE = \frac{9}{2} \rightarrow FC = \frac{10}{2} \rightarrow$

$\rightarrow BC = \frac{10}{2} + C = \frac{27}{2} \checkmark$

$\hat{H} = \hat{H}$ (مقابل قائمه)، $\hat{CFE} = \hat{DEF}$ (مقابل قائمه) $\rightarrow \hat{DHE} \cong \hat{FHC}$

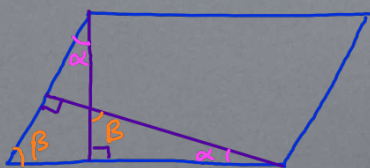


$AB^2 = 5 \times 20, AC^2 = 19 \times 20 \rightarrow \frac{AB^2}{AC^2} = \frac{5 \times 20}{19 \times 20} = \frac{5}{19}$

$\frac{AB}{AC} = \frac{\sqrt{5}}{\sqrt{19}} = \frac{1}{\sqrt{19}} \checkmark$

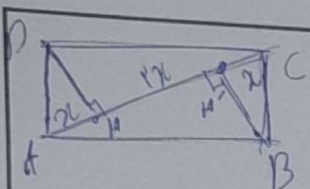
$AB^2 = BH \times BC$
 $AC^2 = CH \times CB$

$\hat{CEF} \cong \hat{ABF}$



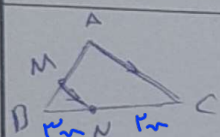
زاویه $\alpha \rightarrow \frac{1}{4} = \frac{1}{x+1}$ زاویه β

$\rightarrow x = 4, AF = x + 1 = 5$



$$\angle H = h = \angle BH', \angle DAH = \alpha \rightarrow \angle ADH = 90 - \alpha \rightarrow \angle HDC = \alpha, \angle DCH = 90 - \alpha \rightarrow \triangle ADH \sim \triangle CDH$$

$$\frac{CH}{h} = \frac{h}{x} \rightarrow CH = \frac{h^2}{x} \rightarrow \frac{h^2}{x} = \frac{h}{x} \rightarrow h = \alpha \sqrt{x} \rightarrow AB = AH + BH \rightarrow \rightarrow AB = \sqrt{5}x \rightarrow S_{\square} = \sqrt{5}x \times 2x^2, S_{\Delta} = \frac{\sqrt{5}x \times x}{2} \rightarrow \frac{S_{\square}}{S_{\Delta}} = 2 \checkmark$$

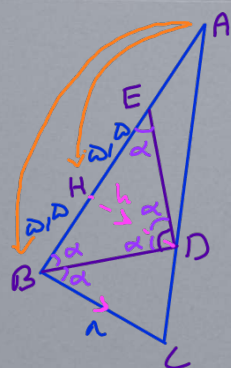


در این شکل AC و NM موازی است

$$\frac{S_{BAC}}{S_{BMN}} = \frac{1}{4} = \frac{\sin B}{\sin B} \times \frac{AB \times CB}{x \times x} \rightarrow AB \times CB = 4x^2$$

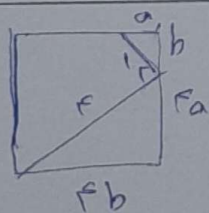
$$\triangle BNM \sim \triangle BAC \rightarrow \frac{BN}{BM} = \frac{1}{2} = \frac{AB}{AB} \rightarrow AB = 2\sqrt{5}x, CB = 2\sqrt{5}x$$

$$\frac{BM}{AM} = \frac{2x}{(2\sqrt{5}-1)x} = \frac{2}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{2}$$



$$h^2 = \Delta_1 \Delta_2 \times \Delta_1 \Delta_2 \rightarrow h = \Delta_1 \Delta_2$$

$$\cos C = \frac{n + \Delta_1 \Delta_2}{11 + n} = \frac{\Delta_1 \Delta_2}{n} \rightarrow n = \frac{11}{\Delta_1 \Delta_2}$$



$$14a^2 + 14b^2 = 4, a^2 + b^2 = 1 \rightarrow 13b^2 = b^2 + 13a^2 \rightarrow 13b^2 = 13a^2$$

$$9a^2 + 9b^2 = 9 \rightarrow 9a^2 + 14a^2 = 9 \rightarrow 23a^2 = 9 \rightarrow a = \frac{3}{\sqrt{23}}, b = \frac{2}{\sqrt{23}}$$

$$a = \frac{3}{\sqrt{23}}, b = \frac{2}{\sqrt{23}} \rightarrow 13b^2 = \frac{16}{23} = \frac{16}{23}$$

$$S = \text{ضلع}^2 = \frac{16}{23} \times \frac{16}{23} = \frac{256}{529} \checkmark$$

$$\text{محور ثقل در وسط است} \rightarrow G \rightarrow BG = 2m, GD = m \rightarrow BD = 3m$$

از A به G وصل کنیم، AE، GD، AGC موازی هستند:

$$GF = \frac{2}{3}m \rightarrow \frac{BD}{FD} = \frac{3m}{\frac{1}{3}m} = 9$$

$$FD = \frac{1}{3}m$$

$$\frac{S_{ABC}}{S_{BMN}} = \frac{\cancel{\frac{1}{r}} AB \times \overset{\Delta}{\cancel{BC}} \times \cancel{\sin B}}{\cancel{\frac{1}{r}} BM \times \underset{\sim}{\cancel{BN}} \times \cancel{\sin B}} = r \rightarrow \frac{AB}{BM} = \frac{r}{\Delta}$$

$$\frac{AB - BM}{BM} = \frac{r - \Delta}{\Delta} = \frac{r}{\Delta}$$

$$\frac{BM}{AM} = \boxed{\frac{\Delta}{r}}$$