امیر صفری

A تکلیف شماره ۲۱

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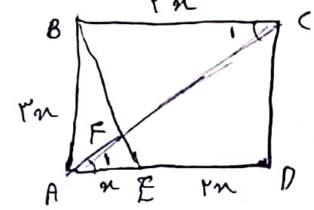

$\overline{BC} \parallel \overline{AD}$ و $\overline{AC}$ مورب $\Rightarrow \hat{A}_1 = \hat{C}_1$

$\hat{F}_1$ و $\hat{F}_2$ متقابل $\Rightarrow \hat{F}_1 = \hat{F}_2$

$\xrightarrow{\text{زز}} \triangle BCF \sim \triangle EAF$

$\frac{BC}{AE} = \frac{3x}{x} = 3 \Rightarrow k$

$\overline{BE} = \sqrt{(3x)^2 + x^2} = \sqrt{10}x \Rightarrow \overline{EF} \in \overline{BC},\ \frac{1}{4}\overline{BC} = \overline{EF} \Rightarrow \overline{EF} = \frac{\sqrt{10}x}{4}$

$\overline{AC} = \sqrt{(3x)^2 + (3x)^2} = 3\sqrt{2}x \Rightarrow \overline{AF} \in \overline{AC},\ \frac{1}{4}\overline{AC} = \overline{AF} \Rightarrow \overline{AF} = \frac{3\sqrt{2}x}{4}$

$\Rightarrow \frac{EF}{AF} = \frac{\frac{\sqrt{10}x}{4}}{\frac{3\sqrt{2}x}{4}} = \boxed{\frac{\sqrt{5}}{3}}$

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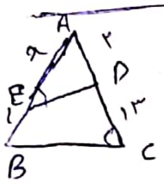

$A\hat{E}D = A\hat{C}B$

$\Rightarrow \hat{A}$ مشترک

$A\hat{E}D = A\hat{C}B$

$\xrightarrow{\text{زز}} \triangle ADE \sim \triangle ABC$

$\frac{AE}{AC} = \frac{AD}{AB} \Rightarrow \frac{x}{15} = \frac{2}{x+1}$

$x^2 + x = 30$

$x^2 + x - 30 = 0$

$(x-5)(x+6) = 0$

$\boxed{x = 5}$ و -6 غ ق ق

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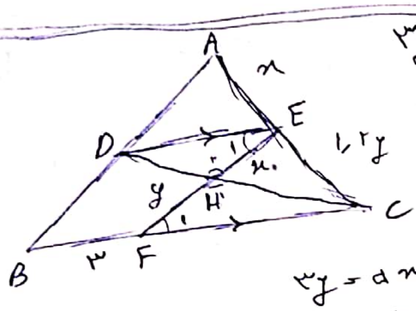

$3y = 5x$

$BC = ?$

H_1, H_2 متقابل $\Rightarrow \hat{H}_2 = \hat{H}_1$

$BC \parallel DE \Rightarrow \hat{E}_1 = \hat{F}_1$ و FE مورب

$\xrightarrow{\text{زز}} \triangle HDE \sim \triangle HFC$

$\frac{3}{5} = \frac{x}{y} = \frac{DE}{CF} \Rightarrow 3z,\ 5z$

$DE \parallel BC \xrightarrow{\text{تالس}} \frac{DE}{BC} = \frac{AD}{AB} = \frac{AE}{AC}$

$\frac{3z}{5z+3} = \frac{x}{1.2y + x} \Rightarrow \frac{3z}{5z+3} = \frac{x}{3x}$

$9z = 5z + 3$

$4z = 3$

$z = \frac{3}{4}$

$3y = 5x$

$1.2y = \frac{6}{5} \times \frac{5}{3}x$

$1.2y = 2x$

$BC = 3 + 5z = 3 + \frac{5 \times 3}{4} = \boxed{\frac{27}{4}}$

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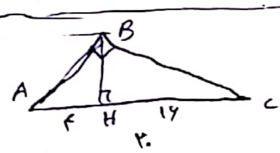

$AB^2 = AH \times AC$ (4×20)

$AB = \sqrt{80} = 4\sqrt{5}$

$BC^2 = CH \times AC$ (16×20)

$BC = \sqrt{320} = 8\sqrt{5}$

$\frac{AB}{BC} = \frac{4\sqrt{5}}{8\sqrt{5}} = \boxed{\frac{1}{2}}$

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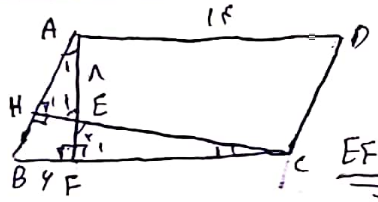

$\hat{E}_1$ و $\hat{E}_2$ متقابل $\Rightarrow \hat{E}_1 = \hat{E}_2$

$90^\circ = \hat{H}_1 = \hat{F}_1$

$\xrightarrow{\text{زز}} \triangle AHE \sim \triangle FEC$

$\Rightarrow \hat{A}_1 = \hat{C}_1$

$\hat{F}_1 = \hat{F}_2 = 90^\circ$

$\xrightarrow{\text{زز}} \triangle ABF \sim \triangle FEC$

$EF = x$

$\frac{AB}{CE} = \frac{BF}{FE} = \frac{AF}{FC} \Rightarrow \frac{6}{x} = \frac{8+x}{14-6} \Rightarrow 48 = 8x + x^2$

$x^2 + 8x - 48 = 0$

$(x+12)(x-4) = 0$

-12 غ ق ق و $x = 4$

$EF = 4$

$AF = 8 + EF = 8 + 4 = 12$

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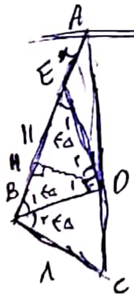

$\hat{B}_1 = \hat{B}_2$

$DH \parallel BC \Rightarrow \hat{B}_2 = \hat{D}_1$ و BD مورب

$\hat{B}_1 + \hat{D}_1 = 90^\circ$

$\hat{B}_1 + \hat{B}_2 = 90^\circ$

$\hat{B}_1 = \hat{B}_2$

$\hat{B}_1 = \hat{B}_2 = \hat{D}_1 = 45^\circ$

$\hat{B}_1 + \hat{E}_1 = 90^\circ$

$45 + \hat{E}_1 = 90^\circ$

$\hat{E}_1 = 45^\circ$

$\hat{E}_1 = \hat{B}_1 \Rightarrow \overline{BD} = \overline{ED}$

$\hat{D} = 90^\circ$

$\overline{EH} = \overline{BH}$

$\frac{11}{2} = 5.5$

$HD^2 = EH \times BH \Rightarrow HD = \sqrt{(5.5)^2} = 5.5$

$DH \parallel BC$

$AE = x$ تالس

$\frac{DH}{BC} = \frac{AH}{AB} \Rightarrow \frac{5.5}{8} = \frac{x + 5.5}{x + 11}$

$5.5x + 60.5 = 8x + 44$

$2.5x = 16.5$

$\boxed{x = 6.6}$

$\boxed{AE = 6.6}$

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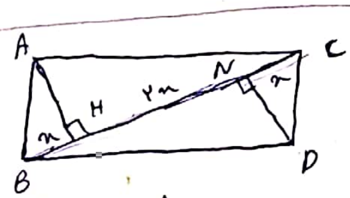

$\triangle ABC \Rightarrow AH^2 = BH \times BC$

$AH^2 = x \times 3x$

$AH = \sqrt{3}x$

مستطیل $S \Rightarrow \triangle ABC\ S2$

$2 \times \frac{\sqrt{3}x \times 4x}{2} = 4\sqrt{3}x^2$

$\triangle NCD\ S$ یا $\triangle ABH\ S = \frac{\sqrt{3}x \times x}{2} = \frac{\sqrt{3}x^2}{2}$

$\frac{S\ ABCD}{S\ \triangle ABH} = \frac{4\sqrt{3}x^2}{\frac{\sqrt{3}x^2}{2}} = 8$

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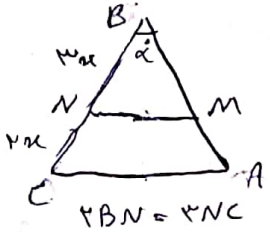

$\frac{S_{ABC}}{S_{ANM}} = \frac{\frac{1}{2}\sin\alpha \cdot BC \cdot AB}{\frac{1}{2}\sin\alpha \cdot BN \cdot BM} = 4$

$\frac{BC \cdot AB}{BN \cdot BM} = \frac{4x}{2x} \cdot \frac{BM+MA}{BM} = 4 \quad \frac{BM+MA}{BM} = \frac{9}{5} \quad 1 + \frac{MA}{MB} = \frac{9}{5}$

$\frac{MA}{MB} = \frac{4}{5} \leftarrow$

$\boxed{\frac{BM}{AM} = \frac{5}{4}}$

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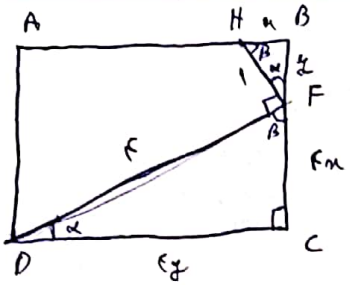

$x^2 + y^2 = 1 \rightsquigarrow x^2 + \frac{14}{9}x^2 = 1 \quad 4y = \frac{14}{5}$

$4x + y = 4y \quad \frac{25}{9}x^2 = 1 \quad$ در نتیجه $(4y)^2 = (\frac{14}{5})^2 = \boxed{\frac{784}{25}}$

$4y = 3x$

$y = \frac{3}{4}x \quad \frac{5}{3}x = 1 \rightsquigarrow x = \frac{3}{5}, y = \frac{4}{5}$

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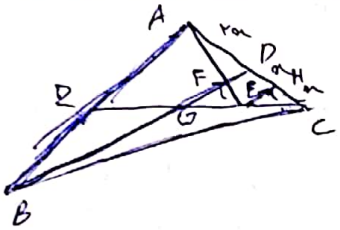

BD مرکز ثقل G $\frac{BD}{FD}$؟ AE ، $EH \parallel BD \rightsquigarrow$ BD میانه $\rightsquigarrow AD = DC$

$GE = EC$ میانه DC و EH.

$HE \parallel BD \xrightarrow[A\hat{E}C]{\text{تالس}} \frac{FD}{EH} = \frac{AD}{AH} = \frac{2x}{x} = 2$

$HE \parallel BD \xrightarrow[G\hat{E}D]{\text{تالس}} \frac{EH}{GD} = \frac{CH}{CD} = \frac{x}{2x} = \frac{1}{2}$

G محل تلاقی $= \frac{GD}{BG} = \frac{1}{2}$ ، $\frac{EH}{BD} = \frac{1}{4}$ $\quad \frac{FD}{BD} = \frac{FD}{EH} \times \frac{EH}{BD} = \frac{1}{2}$

$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

$\boxed{\frac{BD}{FD} = 2}$