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المثلثين $\triangle ABC$ و $\triangle DEF$ لهما مركز ثقل واحد

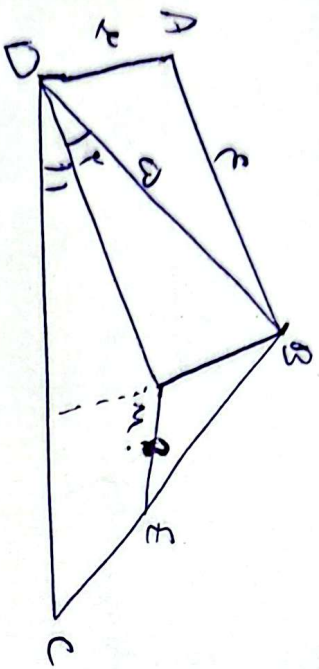
(1)

(1)

$$\frac{S_{\triangle ABC}}{S_{\triangle DEF}} = \frac{BC}{DE} = \frac{AB}{AD} = \frac{1}{2} \Rightarrow \boxed{\frac{1}{4}} \quad \checkmark$$

(2)

(2)



$$\begin{aligned} DC &= a \rightarrow \hat{A} = \hat{A}' \rightarrow \triangle ABC \sim \triangle A'B'C' \rightarrow \frac{BC}{B'C'} = \frac{AB}{A'B'} = \frac{AC}{A'C'} \\ \Rightarrow \frac{BC}{B'C'} &= \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{1}{2} \Rightarrow \boxed{BC = \frac{1}{2} B'C'} \quad \checkmark \end{aligned}$$

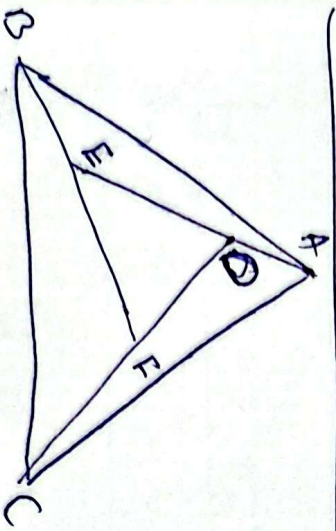
(3)

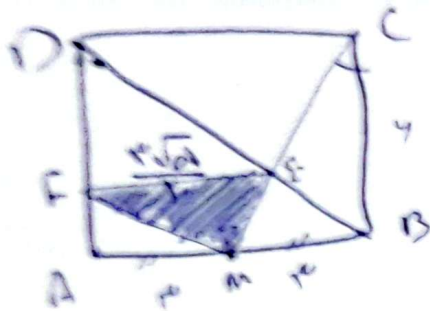
$$\triangle DEF \sim \triangle ABC$$

$$\Rightarrow \frac{DE}{BC} = \frac{EF}{AC} \Rightarrow \frac{1}{2} = \frac{EF}{AC} \Rightarrow AC = \frac{2 \times EF}{1} = \frac{2 \times 1}{1} = \boxed{2} \quad \checkmark$$

(4)

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$$\triangle AMF \cong \triangle MCB \Rightarrow \frac{x}{4} = \frac{m}{x} \rightarrow x = \frac{4}{r}$$

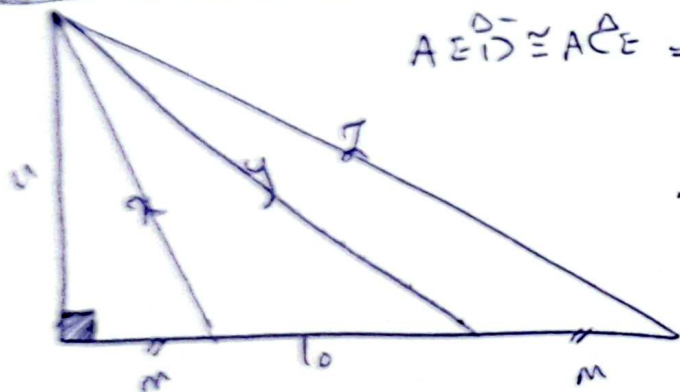
$$MF = \sqrt{4 + \frac{4}{r}} = \frac{4\sqrt{5}}{r}$$

$$\frac{4}{r} = \frac{MC}{\frac{4\sqrt{5}}{r}} \rightarrow MC = 4\sqrt{5} \rightarrow \frac{ME}{4\sqrt{5}} = \frac{x}{4} \rightarrow ME = \sqrt{5}$$

$$\rightarrow S_D = \frac{\sqrt{5} \times 4\sqrt{5}}{2 \times r} = \left(\frac{10}{r}\right) \checkmark$$

امین بنیخاوری و زهره آریکری

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$$\triangle AED \cong \triangle ACE \Rightarrow \frac{x}{10+m} = \frac{4}{2} = \frac{10}{x} \rightarrow m^2 = 100 + 10m$$

$$\Rightarrow |x| + m^2 = 100 + 10m \Rightarrow m^2 - 10m + 100 = 0 \rightarrow (m-5)(m-20) = 0$$

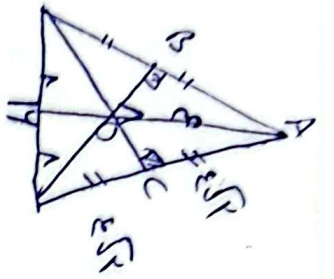
$$\Rightarrow \boxed{m=5} \checkmark \quad \boxed{m=20} \checkmark$$

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$$(EF)^2 = 14 \times FC \rightarrow FC = 14$$

$$(BC)^2 = AC \times FC \Rightarrow (BC)^2 = 14 \times r \rightarrow BC = \sqrt{14 \times r} = 7 \times \sqrt{2} = 7\sqrt{2} \checkmark$$

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$$\triangle ABO \cong \triangle ACO \text{ (SAS)} \Rightarrow \overline{AB} = \overline{AC}$$

$$\Rightarrow OB = OC = \frac{2}{3} \times \sqrt{10} = \frac{4\sqrt{10}}{3}$$

امیدوارم مفید باشد

(1) (2)

$$BC^2 = 4 + 16 = 20 \Rightarrow BC = \sqrt{20} = 2\sqrt{5} \Rightarrow BE = \sqrt{5}$$

$$\tan C_1 = \frac{1}{2} = \frac{\sqrt{5}}{AE} \Rightarrow AE = 2\sqrt{5}$$

$$\Rightarrow (3) = \frac{\sqrt{20} \times \sqrt{5}}{2} = \frac{\sqrt{100}}{2} = \frac{10}{2} = 5$$

$$\tan x = \frac{4}{3} \Rightarrow \cos x = \frac{3}{5}$$

$$\Rightarrow BE = BC \times \frac{BC}{BC} = \frac{BC^2}{BC} = \frac{20}{2} = 10$$

$$(AB)^2 + (BC)^2 = (AC)^2 \Rightarrow \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

$$\frac{4 \times 16}{2} = \frac{64}{2} = 32$$

$$16 = 4 \times 4 \Rightarrow HC = 4 \Rightarrow AE = \frac{4 - 16}{2} = -6$$

$$\frac{DE}{BC} = \frac{AE}{AC} \Rightarrow \frac{DE}{2} = \frac{4}{5} \Rightarrow DE = \frac{8}{5} = 1.6$$