نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۴ ..... کلاس .....

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$x=1 \Rightarrow 3^{A+B}=1 \Rightarrow A+B=0$

$x=2 \Rightarrow 3^{2A+B}=9 \Rightarrow 2A+B=2$

$\Rightarrow A=1, B=-1$

$f(x)=3^{x-1}$

$x=0 \Rightarrow \boxed{f(x)=\frac{1}{3}}$

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$4^x+10=2^{x+2} \Rightarrow 2^{2x}+10=4\times 2^x$

$a^2+10=4a \Rightarrow (a-5)(a-2)=0 \Rightarrow a=5,2$

$\Rightarrow 2^x=5,2$

$2^{x_1+x_2}=2^{x_1}\cdot 2^{x_2}=5\times 2=10$

$\Rightarrow x_1+x_2=\boxed{\log_2 10}$

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$\log_{21}^3=a \Rightarrow \log_{21}^7=1-a$

$147=7\times 21$

$\log_{21}^{147}=1+\log_{21}^7$

$1323=3^2\times 21$

$\log_{21}^{1323}=2+\log_{21}^3$

$(\log_{21}^3)^2+(1+\log_{21}^7)(2+\log_{21}^3)=$

$a^2+(1+1-a)(2+a)=a^2+4-a^2=\boxed{4}$

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$2\log_{(x-1)^2}(1-x)+3\log(1-x)=5 \Rightarrow \log(1-x)=1 \Rightarrow 1-x=10 \Rightarrow x=-9$

$\log_{\frac{x}{3}}=\log_{\frac{9}{3}}=\boxed{2}$

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$\log_{\frac{x}{\sqrt[3]{2}}}=\log_{\frac{\sqrt[3]{4}}{\sqrt[3]{2}}}=\boxed{4}$

$\log_2^{(x^2+2x+4)(x-2)}=3$

$(x^2+2x+4)(x-2)=8$

$\Rightarrow x^3+2x^2+4x-2x^2-4x-8=8$

$\Rightarrow x^3=16 \Rightarrow x=\sqrt[3]{4^3}$

$(x-2)^2 = (2-x)^2$

$\log(2-x) + 2\log(2-x) = 3$

$\Rightarrow \log(2-x) = 1 \Rightarrow 2-x = 10 \Rightarrow x = -8$

$\log_{\sqrt{2}} \frac{-x}{16} = \log_{2^{\frac{1}{2}}} 2^3 = \boxed{6}$

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$x^2 - 2 = 6x \rightarrow x^2 - 6x - 2 = 0 \Rightarrow x = \frac{6 \pm \sqrt{36+8}}{2} = \frac{6 \pm 2\sqrt{11}}{2} = 3 \pm \sqrt{11}$ ①

$x > 2$ ②

$\log_4 \frac{x-2}{4} \overset{①}{\underset{②}{=}} \log_4 \frac{\sqrt{11}}{4} = \boxed{\frac{1}{2}}$

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$\log_{12} 8 = \frac{3\log 2}{2\log 3 + \log 2} \overset{\div \log 3}{\underset{\div \log 3}{=}} \frac{3\log_3 2}{2 + \log_3 2} = \frac{3 \cdot \frac{5}{8}}{2 + \frac{5}{8}} = \frac{\frac{15}{8}}{\frac{21}{8}} = \boxed{\frac{5}{7}}$

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$\log_{12} 4 = \frac{\log 2 + \log 3}{2\log 2 + \log 3} = \frac{1 + \log_2 3}{2 + \log_2 3} = \frac{2,4}{3,4} = \boxed{\frac{12}{17}}$

$\log_2 3 = 2\log_{2^2} 3 = 1,4$

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$a\log 2 - a + b\log 2 = 0 \Rightarrow a(\log 2 - 1) + \log 2^b = 0$

$\Rightarrow \log(\frac{2}{10})^a = \log 2^{-b} \xrightarrow{\text{پتانسیل}} (\frac{2}{10})^a = 2^{-b} \Rightarrow 2^{a+b} = 10^a$

$\Rightarrow 2^{1+\frac{b}{a}} = 10$

$\Rightarrow 2^{\frac{b}{a}} = 5$

$\Rightarrow (\sqrt{2})^{\frac{b}{a}} = \boxed{\sqrt{5}}$

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