نام و نام خانوادگی : علی رابنی — پاسخنامه تکلیف شماره ۲۴ — کلاس یازدهم ۸

۱-

$y = x^2 \qquad f(x) = 3^{Ax+B} \qquad f(x) = 3^{x-1} \qquad 3^{0-1} = \boxed{\frac{1}{3}} \qquad \boxed{\frac{1}{3}}$

$y = 1^2 \Rightarrow (1,1) \qquad 3^{A+B} = 1 \Rightarrow A+B=0 \qquad 2A = 2 \qquad (0, \frac{1}{3})$

$y = 3^2 \Rightarrow (3,9) \qquad 3^{3A+B} = 9 \Rightarrow 3A+B = 2 \qquad A=1 \Rightarrow \boxed{B=-1}$

۲-

$\log_2^{(4^x+15)} = x+3$

$2^{x+3} = 4^x + 15 \Rightarrow 2^x \times 2^3 = 2^{2x} + 15 \Rightarrow 2^{2x} + 15 - 2^x \times 2^3 \xrightarrow{2^x = t} t^2 + 15 - 8t =$

$t^2 - 8t + 15 \longrightarrow (t-3)(t-5)$

$\begin{cases} t = 5 \Rightarrow 2^x = 5 \longrightarrow \operatorname{Log}_2^5 = x \\ t = 3 \Rightarrow 2^x = 3 \longrightarrow \operatorname{Log}_2^3 = x \end{cases} \longrightarrow \operatorname{Log}_2^5 + \operatorname{Log}_2^3 = \boxed{\operatorname{Log}_2^{15}}$

۳-

$(\log_{21}^3)^2 + \log_{21}^{\overset{3\times7^2}{(147)}} \log_{21}^{\overset{9\times3\times7^2}{(1323)}} = (\log_{21}^3)^2 + (\operatorname{Log}_{21}^9 + \overset{1}{\cancel{\operatorname{Log}_{21}^{21}}} + \operatorname{Log}_{21}^7)(\overset{1}{\cancel{\operatorname{Log}_{21}^{21}}} + \operatorname{Log}_{21}^7)$

$(\log_{21}^3)^2 + (\operatorname{Log}_{21}^9 + \operatorname{Log}_{21}^7 + 1)(\operatorname{Log}_{21}^7 + 1) \xrightarrow[\log_{21}^7 = z]{\log_{21}^3 = t} t^2 + (2t + z + 1)(z+1)$

$t^2 + 2tz + 2t + z^2 + z + z + 1 = \underbrace{t^2 + z^2 + 2tz}_{} + 2z + 1 + 2t$

$(t+z)^2 + 2(z+t) + 1 = (\overset{1}{\cancel{\operatorname{Log}_{21}^3 + \operatorname{Log}_{21}^7}})^2 + 2(\operatorname{Log}_{21}^3 + \operatorname{Log}_{21}^7)$

$1 + 2 + 1 = \boxed{4}$

۴-

$\operatorname{Log}(x^2 - 2x + 1) + 3\operatorname{Log}(1-x) = 5 \qquad (x-1)^2 = (1-x)^2$

$\operatorname{Log}(x-1)^2 + 3\operatorname{Log}(1-x) - 5 = 0 \Rightarrow \operatorname{Log}(1-x)^2 + 3\operatorname{Log}(1-x) - 5 = 0$

$2\operatorname{Log}(1-x) + 3\operatorname{Log}(1-x) - 5 = 0$

$5\operatorname{Log}(1-x) - 5 = 0 \qquad \operatorname{Log}^{1-x} = 1$

$\log_3^{-x} = \operatorname{Log}_3^9 = \boxed{2} \qquad \Rightarrow x = -9$

۵-

$\operatorname{Log}_2^{(x^2+2x+4)} + \operatorname{Log}_2^{(x-2)} = 3 \Rightarrow \operatorname{Log}_2^{(x-2)(x^2+2x+4)} = 3 \Rightarrow \operatorname{Log}_2^{x^3-8} = 3 \Rightarrow x^3 - 8 = 8$

$x^3 = 16 \quad x = \sqrt[3]{16}$

$\log_{\sqrt[3]{2}}^{x} = \operatorname{Log}_{\sqrt[3]{2}}^{\sqrt[3]{16}} = \boxed{4}$

6-

$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \qquad (2-x)^3 = 1000$

$2-x = 10 \qquad \boxed{x=-8}$

$\frac{\log (2-x)}{\log\frac{1}{(2-x)^2}} = \log_{10}(2-x)^3 = 3 \qquad \log_{\sqrt{2}} 8 = 2\log_2 8 = \boxed{6}$

7-

$3^{x^2-2} = 81^x = 3^{4x} \qquad \log_6 (x-2)^2 = 1 \qquad \boxed{\frac{1}{2}}$

$x^2-2 = 4x \qquad x^2-4x-2=0 \qquad = 2\log_6 (x-2) = 1$

$(x-2)^2 = 6 \qquad \log_6 (x-2) = \boxed{\frac{1}{2}}$

8-

$\log_3 2 = \frac{5}{8} = \frac{\log 2}{\log 3} = \frac{1}{\log_2 3} = \frac{5}{8} \Rightarrow \log_2 3 = \frac{8}{5}$

$\log_{18} 8 = \frac{\log_2 8}{\log_2 18} = \frac{3}{1+\log_2 9} = \frac{3}{1+2\log_2 3} = \frac{3}{1+\frac{16}{5}} = \frac{15}{21} = \boxed{\frac{5}{7}}$

9-

$\log_4 3 = 0/8$

$\log_{12} 6 = \frac{\log_4 6}{\log_4 12} = \frac{\log_4 2 + \log_4 3}{\log_4 4 + \log_4 3} = \frac{0/5 + 0/8}{1+0/8} = \frac{1/3}{1/8} = \boxed{\frac{13}{18}}$

10-

$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{x=-1} a\log 2 - a + b\log 2 = 0$

$a\log 2 + b\log 2 = a \qquad \log 2^a + \log 2^b - a = 0$

$\log 2(a+b) = a \qquad \log 2^{(a+b)} - \cancel{a} = 0 \Rightarrow \log 2^{(a+b)} - \log 10^a = 0$

$\log\frac{2^{a+b}}{10^a} = 0 \Rightarrow \frac{2^{a+b}}{10^a} = 1 \qquad \Leftarrow \frac{\log 2^{(a+b)}}{\log 10^{(a)}} = 0$

$\frac{2^a \times 2^b}{2^a \times 5^a} = 1 \qquad 2^b = 5^a \qquad \log_5 2^b = a \Rightarrow b\log_5 2 = a$

$(\sqrt{2})^{\frac{b}{a}} = \sqrt{2}^{\frac{b}{b\log_5 2}} = \sqrt{2}^{\log_2 5} = 5^{\frac{1}{2}\log_2 2} \Rightarrow \boxed{\sqrt{5}}$