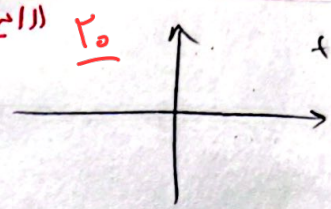
«امیرعلی وحید» «گروه A» ۲۰

۱- (۲)

$3^{Ax+B} = x^2 \rightsquigarrow 3^{3A+B} = 3^2 \rightsquigarrow \begin{cases} 3A+B=2 \\ A+B=0 \end{cases}$

$3^{A+B} = \underbrace{1}_{3^0} \to A+B=0$

$2A=2 \to A=1, B=-1$



$f(x)=3^{x-1}$

$\alpha = \frac{1}{3}$

۲- (۲)

$2^{2x}+15=2^{x+3} \longrightarrow 2^x=t \to t^2-8t+15=0$

$(t-5)(t-3) \Rightarrow t=5,3 \to \begin{cases} 2^x=3 \rightsquigarrow \log_2 3 = x \\ 2^x=5 \rightsquigarrow x=\log_2 5 = x \end{cases} \Rightarrow \log_2 3+\log_2 5 \to \log_2 15$

۳- (۲)

$\log_{21} 3 = a$

$\log_{21} 7 = b$

$\log_{21} 21 = 1 \xrightarrow{\text{پس}} a+b=1 \;,\; 147 = 3\times 7^2 \to \log_{21} 147 = a+2b \to a^2+(a+2b)(a+2)$

$1323 = 3\times 21^2 \rightsquigarrow \log_{21} 1323 = a+2$

$b=1-a$

$a+2b = a+2(1-a) = 2-a \to (2-a)(a+2) = 4-a^2 \to a^2+4-a^2=4$

۴- (۲)

$\underbrace{(x^2-2x+1)}_{(x-1)^2}(1-x)^3 = 10^5 \rightsquigarrow (t^2)(-t)^3 = 10^5 \to -t^5 = 10^5 \rightsquigarrow -t=10$

$x-1=t$

$-x+1=10 \to -x=9$

$x=-9 \to \log_3 3^2 \to 2$

۵- (۲)

$(x^2+2x+4)(x-2) = 2^3$

$x^3+\cancel{2x^2}+\cancel{4x}-\cancel{2x^2}-\cancel{4x}-8=8 \to x^3-16=0 \rightsquigarrow x^3=2^4 \to x=2^{\frac{4}{3}} \to \log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} \to 4$

۶- (۲)

$\dfrac{\frac{(2-x)}{1}}{\frac{1}{(x-2)^2}} = 10^3 \rightsquigarrow \dfrac{\frac{-t}{1}}{\frac{1}{t^2}} = -t^3 = 10^3 \to -t=10 \to t=-10 \to x-2=-10 \to x=-8$

$x-2=t$

$\log_{2^{\frac{1}{2}}} 2^3 \to 6$

۷- (۲)

$3^{x^2-2} = 3^{4x} \longrightarrow x^2-2=4x \longrightarrow x^2-4x-2=0 \rightsquigarrow x=\frac{4\pm\sqrt{16+8}}{2} = \frac{4\pm 2\sqrt{6}}{2} = 2\pm\sqrt{6}$

$\log_6 2+\sqrt{6}-2=\sqrt{6} \to \log_6 6^{\frac{1}{2}} \to \frac{1}{2}$

۸- (۲)

$\log_{18} 2^3 \to 3\log_{18} 2 \to \frac{3}{\log_2 18} \to \frac{3}{\log_2 9+\log_2 2} = \frac{3}{1+2\underbrace{\log_2 3}_{\frac{8}{5}}} \Rightarrow \frac{3}{1+\frac{16}{5}=\frac{21}{5}} \Rightarrow \frac{\frac{\cancel{3}}{1}}{\frac{\cancel{21}\,7}{5}} = \frac{5}{7}$

۹- (۲)

$\dfrac{\log_3 6}{\log_3 12} \Rightarrow \dfrac{\log_3 2+\cancel{\log_3 3}^{1}}{\cancel{\log_3 3}^{1}+2\log_3 2} = \dfrac{1+\cancel{\log_3 2}^{\frac{10}{16}}}{\cancel{2\log_3 2}^{\frac{10}{8}}+1} \Rightarrow \dfrac{\frac{\cancel{26}\,13}{\cancel{16}\,\cancel{2}\,1}}{\frac{18}{\cancel{8}\,1}} = \frac{13}{18}$

$\frac{1}{\log_3 4} = \frac{8}{10}$

$\frac{1}{2\log_3 2} = \frac{8}{10} \to 2\log_3 2 = \frac{10}{8} \to \log_3 2 = \frac{10}{16}$

۱۰- (۲)

$a\log_{10} 2 - a + b\log_{10} 2 = 0$

$\log_{10} 2^a - a + \log_{10} 2^b = 0 \to \underbrace{\log_{10} 2^a + \log_{10} 2^b = \log_{10} 10^a}$

$2^{ab} = 2^a \times 5^a \leftarrow 2^{ab} = 10^a$

$2^b = 5^a \to \sqrt[a]{2^b} = 5 \rightsquigarrow 2^{\frac{b}{a}} = 5$

$2^{\frac{b}{2a}} = \sqrt{5}$

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