پارسا حاتملیری / یازدهم پسر A

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(۱) (۲)

$x=1 \Rightarrow 3^{A+B}=1=3^0 \Rightarrow A+B=0$

$x=3 \Rightarrow 3^{3A+B}=9=3^2 \Rightarrow 3A+B=2$

$A=1,\ B=-1 \Rightarrow f(x)=3^{x-1} \xRightarrow{x=0} f(0)=3^{-1}=\boxed{\frac{1}{3}}$

(۲) (۲)

$\log_2(4^x+15)=x+3 \Rightarrow 15+4^x=2^{x+3} \Rightarrow 15+2^{2x}=2^{x+3}=8\times 2^x$

$\xRightarrow{2^x=t} t^2+15=8t \Rightarrow t^2-8t+15=0 \Rightarrow (t-3)(t-5)=0$

$t=3=2^x \Rightarrow x=\log_2 3$

$t=5=2^x \Rightarrow x=\log_2 5$

$\log_2 3+\log_2 5=\boxed{\log_2 15}$

(۳) (۲)

$(\log_{21}3)^2+\log_{21}(147)\,\log_{21}(1323)=(\log_{21}3)^2+\log_{21}(21\times 7)\,\log_{21}(21^2\times 3)$

$=(\log_{21}3)^2+(1+\log_{21}7)(2+\log_{21}3)=(\log_{21}3)^2+(2-\log_{21}3)(2+\log_{21}3)=\boxed{4}$

$\log_{21}7 = 1-\log_{21}3$

$(2-\log_{21}3)(2+\log_{21}3) = 4-(\log_{21}3)^2$

(۴) (۲)

$\log(1-x)^2+3\log(1-x)=5 \Rightarrow 5\log(1-x)=5 \Rightarrow \log(1-x)=1 \Rightarrow 1-x=10^1=10$

$\Rightarrow \boxed{x=-9} \qquad \log_3(-x)=\log_3 9=\boxed{2}$

(۵) (۲)

$\log_2\big((x^2+2x+4)(x-2)\big)=3 \Rightarrow (x^2+2x+4)(x-2)=8 \Rightarrow x^3-8=8 \Rightarrow x^3=16$

$\Rightarrow \boxed{x=2^{\frac{4}{3}}} \qquad \log_{\sqrt[3]{2}} x=\log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}}=\frac{\frac{4}{3}}{\frac{1}{3}}\log_2 2=\boxed{4}$

(۶) (۲)

$\Rightarrow \log(2-x)-\log(2-x)^{-2}=3 \Rightarrow 3\log(2-x)=3 \Rightarrow \log(2-x)=1 \Rightarrow 2-x=10^1=10$

$(x-2)^2=(2-x)^2$

$\Rightarrow \boxed{x=-8} \qquad \log_{\sqrt{2}}(-x)=\log_{\sqrt{2}}8=\log_{2^{\frac{1}{2}}}2^3=\frac{3}{\frac{1}{2}}\log_2 2=\boxed{6}$

(۷) (۲)

$3^{x^2-2}=3^{4x} \Rightarrow x^2-2=4x \Rightarrow x^2-4x-2=0 \Rightarrow x^2-4x+4=6 \Rightarrow (x-2)^2=6$

$\Rightarrow \begin{cases} x-2=\sqrt{6} & \text{قق} \\ x-2=-\sqrt{6} & \text{غ ق ق} \leftarrow x-2>0 \text{ باید} \end{cases}$

$\log_6(x-2)=\log_6\sqrt{6}=\boxed{\frac{1}{2}}$

(۸) (۲)

$\log_{18}8=\frac{\log_2 8}{\log_2 18}=\frac{3}{1+2\log_2 3}\overset{(I)}{=}\frac{3}{1+\frac{16}{5}}=\frac{3}{\frac{21}{5}}=\frac{15}{21}=\boxed{\frac{5}{7}}$

$\log_3 2=\frac{5}{8} \Rightarrow \log_2 3=\frac{8}{5} \quad (I)$

⑨ $\log_{12}^{6} = \frac{\log_2^6}{\log_2^{12}} = \frac{1+\log_2^3}{2+\log_2^3} \overset{(I)}{=} \frac{1+1/6}{2+1/6} = \frac{7/6}{13/6} = \boxed{\frac{13}{18}}$ ✓

پاسخ تشریحی / پاسخ تستی A

(۲)

$\log_{\sqrt{2}}^{3} = 0/18 \Rightarrow \frac{1}{2}\log_2^3 = 0/18 \Rightarrow \log_2^3 = 1/6$ (I)

⑩ $\overset{n=-1}{\Rightarrow} a\log^2 - a + b\log^2 = 0 \Rightarrow (a+b)\log^2 = a \Rightarrow \log^2 = \frac{a}{a+b} \Rightarrow \log_2^{10} = \frac{a+b}{a}$

(۲)

$\Rightarrow 1+\log_2^5 = 1+\frac{b}{a} \Rightarrow \log_2^5 = \frac{b}{a}$ (I)

$(\sqrt{2})^{\frac{b}{a}} \overset{(I)}{=} (\sqrt{2})^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$ ✓