

$$x=1 \rightarrow y=1, x=3 \rightarrow y=9 \Rightarrow f(1) = m^{A+B} = 1, f(3) = m^{3A+B} = 9$$

$$\rightarrow A+B=0, 3A+B=2 \rightarrow A=1, B=-1$$

$$f(0) = m^B = m^{-1} = \frac{1}{m}$$

$$\log_p (x^k + 10) = x + m \rightarrow p^{x+m} = x^k + 10 \rightarrow p^{x+m} = p^{2x-7} + 10$$

$$\rightarrow p^{x+m} - p^{2x} = 10 \rightarrow p^x (1 - p^x) = 10 \xrightarrow{p^x=t} 1+t-t^2=10 \rightarrow t=3, 7$$

$$p^x = 3 \rightarrow x = \log_p 3, p^x = 7 \rightarrow x = \log_p 7 \rightarrow \log_p 7 + \log_p 3 = \boxed{\log_p 21}$$

$$\left(\frac{\log_p m}{\log_p n} \right)^r + \frac{\log_p m^r}{\log_p n} \times \frac{\log_p m^r}{\log_p n} = \left(\frac{\log_p m}{1+\log_p n} \right)^r + \frac{r+\log_p m}{1+\log_p n} \times \frac{r+\log_p m}{1+\log_p n}$$

$$\xrightarrow{1+\log_p n=t} \left(\frac{t-1}{t} \right)^r + \frac{t+1}{t} \times \frac{t-1}{t} = \frac{t+r}{t} = r$$

$$x^r - r x + 1 = (1-x)^r \rightarrow \log(x^r - r x + 1) + r \log(1-x) = 0 \rightarrow r \log(1-x) = 0$$

$$\rightarrow \log(1-x) = 1 \rightarrow 1-x=10 \rightarrow x=-9$$

$$\rightarrow \log_m^{-9} = \log_m 9 = r$$

$$\log_p (x^r + r x + 1) + \log_p (x-r) = m \rightarrow (x^r + r x + 1)(x-r) = m \rightarrow x^m - 1 = m$$

$$\rightarrow x = \sqrt[m]{m+1} \rightarrow \log_m \frac{x}{\sqrt[m]{m}} = r$$

$$\log_{r-x} - \log \frac{1}{(x-r)^r} = \log_{10} (r-x)(x-r)^r = \log_{10} (r-x)^r = r \rightarrow x = -1$$

$$\rightarrow \log \frac{-x}{\sqrt{r}} = \log \frac{1}{\sqrt{r}} = 4$$

$$r^{x^2-r} = 11^x \rightarrow r^{x^2-r} = r^{rx} \rightarrow x^2 - rx - r = 0 \rightarrow x = \frac{r \pm r\sqrt{4}}{2} = r \pm \sqrt{4}$$

$$\xrightarrow{x > 0} x = r + \sqrt{4} \rightarrow \log \frac{x-r}{4} = \log \frac{\sqrt{4}}{4} = \frac{1}{r}$$

$$\log_{1\lambda} \frac{1}{\lambda} = \frac{\log \frac{1}{\lambda}}{\log 1\lambda} = \frac{r \log \frac{1}{\lambda}}{r + \log \frac{1}{\lambda}} = \frac{\frac{10}{\lambda}}{\frac{r}{\lambda}} = \frac{10}{r}$$

$$\log_{1r} \frac{4}{1r} = \frac{\log \frac{4}{r}}{\log 1r} = \frac{\log r + \log \frac{4}{r}}{\log r + \log r} = \frac{1, r}{1, \lambda} = \frac{1r}{1\lambda}$$

$$x = -1 \rightarrow a \log r - a + b \log r = 0 \rightarrow (a+b) \log r = a, b \log r = a(1 - \log r)$$

$$\xrightarrow{\text{divide}} \frac{b}{a} = \frac{1 - \log r}{\log r} = \log \frac{a}{r} \rightarrow (\sqrt{r})^{\frac{b}{a}} = \sqrt{r}^{\log \frac{a}{r}} = a^{\log \frac{\sqrt{r}}{r} \frac{1}{r}}$$

$$= \sqrt{a}$$