شایان رومی‌زادن

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$a \,|\, \frac{1}{1} \qquad b \,|\, \frac{3}{9} \qquad f(3) = 3^{3A+B} = 3^{2}$

$f(1) = 3^{A+B} = 3^{0}$

$\rightarrow 3A+B=2,\ \rightarrow A+B=0 \ \} \longrightarrow 2A=2 \rightarrow A=1 \rightarrow B=-1$

$f(x) = 3^{x-1} \rightarrow 3^{0-1} = \frac{1}{3}$

$\Rightarrow y=?,\ x=0 \rightarrow f(0) = 3^{0-1} = \frac{1}{3}$

$\rightarrow \boxed{(0, \frac{1}{3})}$ ✓

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$4^{x} + 15 > 0 \rightarrow$ همواره برقرار

$2^{x+3} = 2^{2x} + 15 \xrightarrow{2^{x}=t} t^{2} - 8t + 15 = 0 \rightarrow (t-3)(t-5) = 0$

	3		5	
+	0	−	0	+

$2^{x+3} = 2^{x} \times 2^{3} = 8 \times 2^{x} = 8t$

$\rightarrow t=3 \rightarrow 2^{x}=3 \rightarrow x = \log_{2} 3$

$t=5 \rightarrow 2^{x}=5 \rightarrow x = \log_{2} 5$

$\rightarrow \log_{2} 3 + \log_{2} 5 = \boxed{\log_{2} 15}$ ✓

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147	3
49	7
7	7
1	

1323	3
441	3
147	3
49	7
7	7
1	

$\log_{21} 3 = m, \quad \log_{21} 7 = n$

$= m^{2} + (\log_{21} 3 + 2\log_{21} 7)(3\log_{21} 3 + 2\log_{21} 7) = m^{2} + (m+2n)(3m+2n)$

$= 4m^{2} + 4n^{2} + 8mn = 4(m+n)^{2} = 4(\log_{21} 3 + \log_{21} 7)^{2} = 4(\log_{21} 21)^{2} = \boxed{4}$ ✓

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$(x^{2} - 2x + 1) = (x-1)^{2} \qquad 1-x > 0 \rightarrow |\sqrt{(x-1)^{2}}| = 1-x$

$= \log (1-x)^{2} + 3\log (1-x) = 5 \xrightarrow{\log (1-x) = t} 2t + 3t = 5 \rightarrow t = 1 \rightarrow \log_{10} (1-x) = 1 \rightarrow 1-x = 10 \rightarrow x = -9$

$\log_{3} (1-x) = \log_{3} 9 = \boxed{2}$ ✓

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$(x^{2} + 2x + 4) = \frac{x^{3} - 8}{x - 2}$

$= \log_{2} (x^{3}-8) - \log_{2} (x-2) + \log_{2} (x-2) = \log_{2} (x^{3}-8) = 3 \rightarrow x^{3} - 8 = 8 \rightarrow x = 2\sqrt[3]{2}$

$\log_{\sqrt[3]{2}} 2\sqrt[3]{2} = \log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = \frac{\frac{4}{3}}{\frac{1}{3}} \log_{2} 2 = \boxed{4}$ ✓

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$= \log^{2-x} - \log^{(2-x)^{-2}} = 3\log^{2-x} = 3 \rightarrow \log_{10}^{2-x} = 1 \rightarrow 2-x = 10 \rightarrow x = -8$ ✓

$\log_{\sqrt{2}}^{(-(-8))} = \log_{2^{\frac{1}{2}}}^{2^3} = 6\log_2^2 = \boxed{6}$ ✓

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$3^{x^2-2} = 3^{4x} \rightarrow x^2 - 2 = 4x \longrightarrow x^2 - 4x - 2 = 0 \rightarrow x = \frac{4 \pm \sqrt{16+8}}{2}$

$x = \frac{4+2\sqrt{6}}{2} = 2+\sqrt{6} \longrightarrow \log_6^{x-2} = \log_6^{2+\sqrt{6}-2} = \log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$ ✓

$x = \frac{4-2\sqrt{6}}{2} < 0$ غ ق ق

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$\log_{18}^{8} = \frac{\log_{27}^{8}}{\log_{27}^{18}} = \frac{\frac{3}{3}\log^2_3}{\frac{1}{3}\log^2_3 + \frac{2}{3}\log^3_3} = \frac{\frac{3}{8}}{\frac{3}{24} + \frac{2}{3}} = \frac{\frac{3}{8}}{\frac{21}{24}} = \boxed{\frac{3}{7}}$ ✓

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$\log_{12}^{6} = \frac{\log_4^6}{\log_4^{12}} = \frac{\frac{1}{2}\log^2_2 + \log^3_4}{\log^3_4 + \log^4_4} = \frac{\frac{1}{2} + \frac{5}{10}}{\frac{5}{10} + 1} = \frac{\frac{13}{10}}{\frac{15}{10}} = \boxed{\frac{13}{15}}$ ✓

۹ (۲)

$x = -1 \rightarrow (a\log^2) + (-a) + b\log^2 = 0 \rightarrow \log^{2^a} + \log^{2^b} - \log_{10}^{10^a} = 0$

$(a+b)\log^2 = a$

$\log^{\frac{2^{a+b}}{10^a}} = 0 \rightarrow \frac{2^{a+b}}{10^a} = 1 \rightarrow 2^a \times 2^b = 2^a \times 5^a \rightarrow 2^b = 5^a \longrightarrow \log_2^5 = b \rightarrow a\log_2^5 = b$

$\rightarrow \frac{b}{a} = \log_2^5 \longrightarrow \sqrt{2}^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$ ✓

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