رایان حصاریان — یازدهم A — تکلیف ۲۴ — نام خدا

① (۲)

$M\Big|_1^1 \; N\Big|_9^3 \rightarrow 1 = 3^{A+B} \;/\; 9 = 3^{3A+B}$

$-A = B \qquad 3A + B = 2$

$\left.\begin{matrix} 2A = 2 \rightarrow \boxed{A = 1} \\ B = -1 \end{matrix}\right\} \quad f(x) = 3^{x-1}$

$f(0) = 3^{-1} = \boxed{\frac{1}{3}}$ ✓

② (۲)

$2^{n+3} = 2^n \times 2^3 = 2^{2n} + 10 = (2^n)^2 - 8(2^n) + 10 = 0 \Rightarrow (2^n - 2)(2^n - 5) = 0$

$2^n = 2 \Rightarrow \log_2^2 = n_1 \;,\; 2^n = 5 \Rightarrow \log_2^5 = n_2 \Rightarrow n_1 + n_2 = \boxed{\log_2^{10}}$ ✓

③ (۲)

$\log_{21}^{(3 \times 7 \times 2)} \times \log_{21}^{(7 \times 3)} = 1 + \log_{21}^{3} \times 2 + \log_{21}^{3} = (2 + \log_{21}^{3})(2 - \log_{21}^{3})$

$4 - (\log_{21}^{3})^2 \rightarrow (\log_{21}^{3})^2 + 4 - (\log_{21}^{3})^2 = \boxed{4}$ ✓

④ (۲)

$\log_{10}^{(1-x)^2(1-x)^3} = 3 \rightarrow 10^3 = (1-x)^5 \rightarrow 1 - x = 10 \rightarrow x = -9 \;/\; \log_3^9 = \boxed{2}$ ✓

⑤ (۲)

$\log_2^{(x-2)(x^2+3x+6)} = 2 \longrightarrow x^3 - 2^3 = 2^3 \rightarrow x^3 = 16 \rightarrow x = 2^{\frac{4}{3}} \rightarrow \log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}}$

$\rightarrow 4 \log_2^2 = \boxed{4}$ ✓

⑥ (۲)

$\log_{(2-x)}^{(2-x)^{-2}} - \log_{(2-x)}^{(2-x)^3} = 0 \rightarrow -2\log_{(2-x)}^{(2-x)} = 3 \rightarrow 10^1 = 2 - x \rightarrow x = -1 \rightarrow \log_{2^{\frac{1}{2}}}^{3} = \boxed{9}$ ✓

$$x^{\log_x 3} = 3^{\log_3 x} \Rightarrow x^2 - 2 = 4x \rightarrow x^2 - 4x - 2 = 0 \rightarrow \frac{4 \pm \sqrt{16+8}}{2} = 2 \pm \sqrt{6}$$

$x > 2 \begin{cases} 2+\sqrt{6} \checkmark \\ 2-\sqrt{6} \times \end{cases} \Rightarrow \log_4 x = \log_{4^{\frac{1}{2}}} (2+\sqrt{6}) = \boxed{\frac{1}{2}}$ ✓

(2) (1)

$$\log_{2^3} \frac{2^2 \cdot 3^3}{\sqrt[3]{2}} = \frac{1}{\frac{1}{3}\log_2 \frac{2^2 \cdot 3^3}{2^{\frac{1}{3}}}} = \frac{1}{\frac{1}{3}(\log_2 2^2 + \log_2 3^3 + \log_2 2^{\frac{1}{3}})} = \frac{1}{\frac{1}{3}(2+3+\frac{1}{3})} = \frac{1}{\frac{1}{3}(\frac{16}{3})}$$

$$= \frac{1}{\frac{16}{9}} = \boxed{\frac{9}{16}}$$ ✓

(2) (9)

$$\log_{2^3} 2^4 = 2 \log_2 2^4 = \frac{16}{8} \Rightarrow \log_2^2 3 = \frac{16}{12} \rightarrow \log_2 3 = 1.6 \rightarrow \log_2 3 + \log_2 2 = \log_2 4 = 2$$

$$\log_{12} 4 = \frac{1}{\log_4 2 \times 4} = \frac{1}{\log_4^2 2 + \log_4^2 4} = \frac{1}{1 + \frac{16}{24}} = \boxed{\frac{13}{11}}$$ ✓

(2) (10)

$$a \log_2 2 - a + b \log_2 2 = \log_2^2 (a+b) - a = 0 \rightarrow \log_2 2 = \frac{a}{a+b}$$

$$b \log_2^{10} 2 = \frac{a+b}{a} \rightarrow \log_2^{10} 2 - 1 = \log_2^{10} 2 - \log_2^2 2 = \log_2^8 2 = \frac{b}{a}$$

$$\rightarrow (2^{\frac{1}{2}})^{\frac{b}{a}} = 2^{\log_2^{\sqrt{8}} 2} = \sqrt{8}$$ ✓