امیرصفری ۱۱ تجربی A تکلیف شماره ۲۴

۱-

$x^2 = 3^{Ax+B} \xrightarrow{\text{دو نقطه دارد}} \begin{cases} (1)^2 = 3^{A+B} \Rightarrow A+B=0 \\ (3)^2 = 3^{3A+B} \Rightarrow 3A+B=2 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$

$3^{x-1} \xrightarrow[x=0 \text{ یعنی}]{\text{تلاقی با محور } y} 3^{-1} = \boxed{\frac{1}{3}}$ $\quad (0, \frac{1}{3})$

۲-

$\log_2 (4^x+15) = x+3$ $\quad 2^{x+3} = 4^x + 15$

$\underbrace{2^x}_{t}(\underbrace{2^x}_{t} - \underbrace{2^3}_{8}) = -15$

$t(t-8) = -15$

$t^2 - 8t + 15 = 0$

$(t-3)(t-5)$

$t = 3, 5$

$2^x = 3$ $\quad \boxed{x = \log_2 3}$

$2^x = 5$ $\quad \boxed{x = \log_2 5}$

$x_1 + x_2 = \log_2 5 + \log_2 3 = \boxed{\log_2 15}$

۳-

$(\log_{21} 3)^2 + [\log_{21} 7 + \underbrace{\cancel{\log_{21} 21}}_{1}] \times [\underbrace{2\cancel{\log_{21} 21}}_{2} + \log_{21} 3] = ?$ $\quad \log_{21} 3 = t$

$\log_{21} 7 = \log_{21} \frac{21}{3} = \underbrace{\cancel{\log_{21} 21}}_{1} - \log_{21} 3 = 1 - \log_{21} 3$

$\cancel{t^2} + \underbrace{(2 \cancel{-t)(2+t})}_{4-\cancel{t^2}} = 4$

۴-

$\log(x^2-2x+1) + 3\log(1-x) = 5$ $\quad 2\log(1-x) + 3\log(1-x) = 5$

$\log(1-x)^2 + 3\log(1-x) = 5$ $\quad \log(1-x) = 1 \Rightarrow 1-x = 10,\ x = -9 \Rightarrow \log_3 -(-9) = 2$

۵-

$\log_2 (x^2+2x+4) + \log_2 (x-2) = 3 \Rightarrow 3 = \log_2 8$ $\quad \log_{\sqrt[3]{2}} \sqrt[3]{16} = \log_2 16 = 4$

$(x^2+2x+4)(x-2) = 8 \Rightarrow \begin{cases} x^3 - 8 = 8 \\ x^3 = 16 \end{cases} \Rightarrow x = \sqrt[3]{16}$

۶-

$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \Rightarrow \log_{10} 1000 = 3$

$(2-x) \times (x-2)^2 = (2-x)(2-x)^2 = (2-x)^3$

$\Rightarrow (2-x)^3 = 1000,\ 2-x = 10,\ x = -8$ $\quad \log_{\sqrt{2}} -(-8) = 6$

۷-

$3^{x^2-2} = 3^{4x}$ $\quad x^2 - 4x - 2 = 0$

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4 \pm \sqrt{24}}{2} = 2 \pm \sqrt{6}$

$2 - \sqrt{6} < 0$ ✗ غ‌ق‌ق

$2 + \sqrt{6} > 0$ ✓

$\log_6 (x-2) = \log_6 \sqrt{6} = \frac{1}{2}$

$$\text{۸-}\qquad \log_{۳}^{۲}=\frac{۵}{۸}\qquad \log_{۱۸}^{۸}=\frac{\log_{۳}^{۲^{۳}}}{\log_{۳}^{۱۸}}=\frac{\cancel{۳\log_{۳}^{۲}}\;\;۳\times\frac{۵}{۸}}{\underbrace{\cancel{۲\log_{۳}^{۳}}}_{۲}+\underbrace{\cancel{\log_{۳}^{۲}}}_{\frac{۵}{۸}}}=\frac{\frac{۱۵}{۸}}{\frac{۲۱}{۸}}=\frac{۱۵}{۲۱}=\boxed{\frac{۵}{۷}}$$

$$\text{۹-}\qquad \log_{۴}^{۳}=۰/۸\qquad \log_{۱۲}^{۶}=\frac{\log_{۴}^{۶}}{\log_{۴}^{۱۲}}=\frac{\overbrace{\cancel{\log_{۴}^{۲}}}^{\frac{۱}{۲}}+\overbrace{\cancel{\log_{۴}^{۳}}}^{۰/۸}}{\underbrace{\cancel{\log_{۴}^{۴}}}_{۱}+\underbrace{\cancel{\log_{۴}^{۳}}}_{۰/۸}}=\frac{۰/۵+۰/۸}{۱+۰/۸}=\frac{۱/۳}{۱/۸}=\boxed{\frac{۱۳}{۱۸}}$$

$$\text{۱۰-}\qquad a\log ۲\,(-۱)^{۲}+(-۱)a+b\log ۲=۰$$

$$a\log ۲-a+b\log ۲=۰$$

$$a\log ۲-a=-b\log ۲\;\rightarrow\;-a(-\quad +۱)=-b\log ۲\;\rightarrow\;a(۱-\log ۲)=b\log ۲\;\rightarrow\;a(\log^{۱۰}-\log^{۲})=b\log^{۲}$$

$$a\log^{۵}=b\log^{۲}$$

$$\frac{b}{a}=\log_{۲}^{۵}$$

$$(\sqrt{۲})^{\frac{b}{a}}=(\sqrt{۲})^{\log_{۲}^{۵}}=۵^{\log_{۲}^{\sqrt{۲}}}=(۵)^{\frac{۱}{۲}}=\boxed{\sqrt{۵}}$$