لادن مکلیان یازدهم ۵

۱۹٫۵

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$x=0 \Longrightarrow y = 1 - \log_c^{-b} = ۲ \Rightarrow b = -\frac{1}{c}$; $c>0 ,\ c \neq 1$

$b + c = -\frac{۳}{۲} \Rightarrow -\frac{1}{c} + c = -\frac{۳}{۲} \Rightarrow c^۲ + \frac{۳}{۲}c - 1 = 0$ (۲)

$\Rightarrow c^۲ + ۳c - ۴ = 0 \Rightarrow c = \frac{1}{۲} ,\ c = -\frac{۴}{۲} = -۲$ $\Big\}$ $c = \frac{1}{۲} ,\ b = -۲$ ✓

$x = -1/۵ \Longrightarrow y = 1 - \log_c^{-1/۵a-b} = 0 \Rightarrow \log_c^{-1/۵a-b} = 1 \Rightarrow -1/۵a - b = c$

$\Rightarrow -1/۵a + 1/۵ = 0 \Rightarrow a = 1$ ✓ $\Rightarrow (a+c)b = (1+\frac{1}{۲})(-۲) = -۳$ ✓

۳- (۲)

$x=0 \Longrightarrow y = c + \log_d^b = ۲ \Rightarrow -\log_d^b + ۲ = c$

$x = ۲/۴ \Longrightarrow y = c + \log_d^{-۲/۴a+b} = 0 \Rightarrow ۲ - \log_d^b + \log_d^{۲/۴a+b} = 0 \Rightarrow \log_d^{۲/۴a+b} = \log_d^b - ۲$

$\Rightarrow \log_d^{۲/۴a+b} = \log_d^{\frac{b}{d^۲}} \Rightarrow ۲/۴a + b = \frac{b}{۲۵} \Rightarrow \frac{a}{b} = -\frac{۲}{۵}$ ✓

۲- (۲)

$x=0 \Rightarrow f(0) = 1 + c \times ۳^a = \frac{۲}{۳} \Rightarrow c \times ۳^a = -\frac{1}{۳}$

$x=1 \Rightarrow f(1) = 1 + c \times ۳^{a+b} = 0 \Rightarrow c \times ۳^a \times ۳^b = -1 \Rightarrow (-\frac{1}{۳})۳^b = -1 \Rightarrow b = 1$

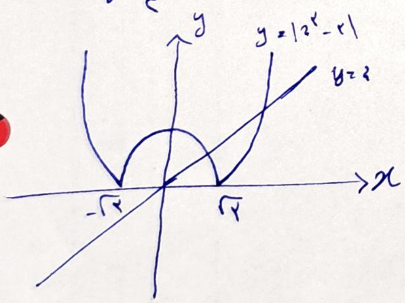
$x=-1 \Longrightarrow f(-1) = 1 + c \times ۳^{a-b} = 1 + \frac{c \times ۳^a}{۳^b} = 1 + \frac{-\frac{1}{۳}}{۳} = 1 - \frac{1}{۹} = \frac{۸}{۹}$ ✓

۴- (۲)

$f(x) = \log_۴(|x^۲ - ۲| - x) \Rightarrow |x^۲ - ۲| - x > 0 \Rightarrow |x^۲ - ۲| > x \Rightarrow x^۲ - ۲ > x \Rightarrow x \geq \sqrt{۲} ,\ x \leq -\sqrt{۲}$

$\Downarrow$

$x^۲ - ۲ < -x \Rightarrow -\sqrt{۲} < x < \sqrt{۲}$



$x^۲ - ۲ > x \Rightarrow x^۲ - x - ۲ > 0 \Rightarrow (x+1)(x-۲) > 0$

$\Rightarrow x \in [(-\infty, -\sqrt{۲}] \cup [\sqrt{۲}, +\infty)]$

-1		۲	
+	−	−	+

$x^۲ - ۲ < x \Rightarrow x^۲ - x - ۲ < 0 \Rightarrow (x-1)(x+۲) < 0$

-۲		1	
+	−	−	+

$\Rightarrow (-\sqrt{۲}, 1) \Longrightarrow D_f = (-\infty, 1) \cup (۲, +\infty)$ ✓

۵- (۲)

$\overset{x=1}{\Longrightarrow} y(1) = -1 - 3 + 8 = 4 \Rightarrow f(1) = 2 + 2^{b-a} \Rightarrow 2^{b-a} = 2 \Rightarrow b - a = 1$

$f^{-1}(10) = -1 \Rightarrow f(-1) = 10$, $f(-1) = 2 + 2^{b+a}$ $\Rightarrow 2 + 2^{b+a} = 10 \Rightarrow 2^{b+a} = 8 \Rightarrow b + a = 3$

$\Rightarrow b = 2, a = 1$

$2b - a = 4 - 1 = 3$

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۶ (۲)

$x = 1 \Rightarrow 1 - 1 = -2 + \left(\frac{1}{2}\right)^{A+B} \Rightarrow \left(\frac{1}{2}\right)^{A+B} = 2 \Rightarrow A + B = -1$

$x = 2 \Rightarrow 4 - 2 = -2 + \left(\frac{1}{2}\right)^{2A+B} \Rightarrow \left(\frac{1}{2}\right)^{2A+B} = 4 \Rightarrow 2A + B = -2$

$\Rightarrow A = -1, B = 0$

$\Rightarrow f(x) = -2 + \left(\frac{1}{2}\right)^{-x} \underset{x=3}{\Rightarrow} f(3) = -2 + 8 = 6$

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۷ (۱/۷۵)

$\left(\frac{8}{9}\right)^{\frac{t}{60}} m = \frac{1}{6} m \Rightarrow \left(\frac{t}{60}\right) \log \frac{8}{9} = \log \frac{1}{6} \Rightarrow \frac{t}{60} \log \frac{9}{8} = \log 6$

$\frac{t}{60} = \frac{\log 6}{\log \frac{9}{8}} = \frac{\log_5 6}{\log_5 \frac{9}{8}} = \frac{\log_5 2 + \log_5 3}{2\log_5 3 - 3\log_5 2} \Rightarrow \frac{\frac{1}{2.4} + \frac{1}{1.4}}{\frac{2}{1.4} - \frac{3}{2.4}} = \frac{3.8}{0.6} = \frac{19}{3}$ ساعت

$\frac{19}{3} \times 60 = 380\ \text{min}$

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۸ (۲)

$m \left(\frac{7}{8}\right)^{\frac{t}{7}} = \frac{1}{7} m \Rightarrow \left(\frac{8}{7}\right)^{\frac{t}{7}} = 7 \Rightarrow \frac{t}{7} \cdot \log \frac{8}{7} = \log 7$

$\Rightarrow \frac{t}{7} = \frac{\log 7}{\log \frac{8}{7}} = \frac{\log_2 7}{\log_2 \frac{8}{7}} = \frac{\log_2 7}{3\log_2 2 - \log_2 7} = \frac{\frac{1}{0.6}}{\frac{1.8 - 1.6}{(1.6)(0.6)}} = \frac{1.6}{0.2} = 8$

$\Rightarrow \frac{t}{7} = 8 \Rightarrow t = 56$ روز

۹- (۲)

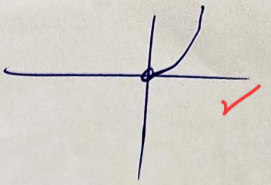
در هر مرحله $\frac{24}{25}$ می شود

$\frac{100}{100} \times \left(\frac{24}{25}\right)^t = \frac{1}{2} \times \frac{100}{100} \Rightarrow \left(\frac{96}{100}\right)^t = \frac{1}{2}$

$(t) \log \frac{96}{100} = \log \frac{1}{2} \Rightarrow t(\log 96 - 2) = -\log 2 \Rightarrow t(5\log 2 + \log 3 - 2) = -\log 3$

$t(1.5 + 0.48 - 2) = -0.48 \Rightarrow t = 24$ روز

الف) $y = 4^{\log_2 x} = 2^{\log_2 x^2} = x^2 \xrightarrow{x>0} D_y = (0, +\infty)$



-10

(۱۷۵)

ب) $y = \log x^2 \Rightarrow x^2 > 0 \Rightarrow x \neq 0 = 2\log x \Rightarrow$

$D_y = R - \{0\}$

به دامنه توجه کن!