پاسخ تشریحی - یازدهم پسر A

① $x=0 \Rightarrow y = 1 - \log_c^{-b} = 2 \Rightarrow \log_c^{-b} = -1 \Rightarrow b = \frac{-1}{c}$ (I) ($c>0$ (II))

$b + c = -\frac{3}{2} \overset{(I)}{\Rightarrow} \frac{-1}{c} + c = -\frac{3}{2} \Rightarrow c^2 + \frac{3}{2}c - 1 = 0 \Rightarrow 2c^2 + 3c - 2 = 0 \xrightarrow{\text{ریشه}} c^2 + 2c - 4 = 0$

$\Rightarrow \begin{cases} c = \frac{1}{2} \\ c = -\frac{3}{2} = -2 \text{ غ ق ق} \end{cases} \overset{(II)}{\Rightarrow} \boxed{c = \frac{1}{2}} \overset{(I)}{\Rightarrow} \boxed{b = -2}$

$x = -1/8 \Rightarrow y = 1 - \log_c^{-1/8a - b} = 0 \Rightarrow \log_c^{-1/8a-b} = 1 \Rightarrow -1/8a - b = c \overset{\text{جایگذاری}}{\Rightarrow} -1/8a + 2 = \frac{1}{2} \Rightarrow -1/8a = -1/5 \Rightarrow \boxed{a = +1}$

$(a+c)b = (1 + \frac{1}{2})(-2) = -3$ ✓ (۴)

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② $x=0 \Rightarrow f(0) = 1 + c \times 2^a = \frac{1}{4} \Rightarrow c \times 2^a = -\frac{1}{4}$ (I)

$x=1 \Rightarrow f(1) = 1 + c \times 2^{a+b} = 0 \Rightarrow c \times 2^a \times 2^b = -1 \overset{(I)}{\Rightarrow} (\frac{-1}{4})(2^b) = -1 \Rightarrow 2^b = 4 \Rightarrow \boxed{b = 2}$ (II)

$x=-1 \Rightarrow f(-1) = 1 + c \times 2^{a-b} = 1 + \frac{c \times 2^a}{2^b} \overset{(I)(II)}{=} 1 + \frac{-\frac{1}{4}}{4} = 1 - \frac{1}{16} = \frac{15}{16}$ ✓

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③ $x=0 \Rightarrow y = c + \log_d^b = 2 \Rightarrow \boxed{\log_d^b + 2 = c}$ (I)

$x=2,2 \Rightarrow y = c + \log_d^{2,2a+b} = 0 \overset{(I)}{\Rightarrow} 2 - \log_d^b + \log_d^{2,2a+b} = 0 \Rightarrow \log_d^{2,2a+b} = \log_d^b - 2$

$\Rightarrow \log_d^{2,2a+b} = \log_d^{\frac{b}{25}} \Rightarrow 2,2a + b = \frac{b}{25} \Rightarrow 2,2a = \frac{24}{25} b \Rightarrow a = -\frac{2}{5} b \Rightarrow \boxed{\frac{a}{b} = -\frac{2}{5}}$ ✓

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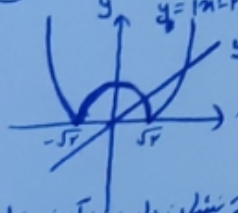
④ $f(x) = \log_2^{(|x^2-2|-x)} \Rightarrow |x^2-2| - x > 0 \Rightarrow |x^2-2| > x \Rightarrow \begin{cases} x^2 - 2 > x & x \geq \sqrt{2} \text{ یا } x < -\sqrt{2} \text{ (I)} \\ x^2 - 2 < -x & -\sqrt{2} < x < \sqrt{2} \text{ (II)} \end{cases}$



(برای بهتر نشان دادن مراحل حل، نمودار کشیدیم.)

$x^2 - 2 > x \Rightarrow \underbrace{x^2 - x - 2}_{g(x)} > 0 \Rightarrow (x+1)(x-2) > 0$

x	$-\infty$	-1	2	$+\infty$
g(x)	+	0	−	+

$\overset{(I)}{\Rightarrow} x \in (-\infty, -1] \cup [2, +\infty)$

$x^2 - 2 < x \Rightarrow \underbrace{x^2 + x - 2}_{h(x)} < 0 \Rightarrow (x-1)(x+2) < 0$

x	-2	1	$+\infty$
h(x)	+	0	−

$\overset{(II)}{\Rightarrow} (-\sqrt{2}, 1)$

$\overset{\cup}{\Rightarrow} \boxed{D_f = (-\infty, 1) \cup (2, +\infty)}$ ✓

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⑤ $x=1 \Rightarrow g(1) = -1 - 2 + 8 = 5 = f(1) = 2 + 2^{b-a} \Rightarrow 2^{b-a} = 3 \Rightarrow \boxed{b - a = 1}$

$f^{-1}(10) = -1 \Rightarrow f(-1) = 10$

$f(-1) = 2 + 2^{b+a}$ $\Rightarrow 2 + 2^{b+a} = 10 \Rightarrow 2^{b+a} = 8 = 2^3 \Rightarrow \boxed{b + a = 3}$

$\Rightarrow 2b = 4 \Rightarrow \boxed{b = 2} \Rightarrow \boxed{a = 1}$

$\Rightarrow 2b - a = 4 - 1 = 3$ ✓ (۳)

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⑥ $x=1 \Rightarrow -1 = -2 + (\frac{1}{4})^{A+B} \Rightarrow (\frac{1}{4})^{A+B} = 2 \Rightarrow \begin{cases} A + B = -1 \\ 2A + B = -2 \end{cases}$

$\boxed{A = -1} \Rightarrow \boxed{B = 0}$ $\Rightarrow f(x) = -2 + (\frac{1}{4})^{-x}$

$x=3 \Rightarrow f(3) = -2 + 8 = 6$ ✓ (۶)

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⑦ $(\frac{1}{9})^{\frac{t}{60}} = \frac{1}{3} \Rightarrow (\frac{t}{60}) \log^{\frac{1}{9}} = \log^{\frac{1}{3}} \overset{\times(-1)}{\Rightarrow} -(\frac{t}{60}) \log^{\frac{1}{9}} = -\log^{\frac{1}{3}} \Rightarrow (\frac{t}{60}) \log^{\frac{1}{3}} = \log^3$

$\frac{t}{60} = \frac{\log^3}{\log^{\frac{1}{9}}} = \frac{\log^3_3}{\log^{\frac{1}{3}}_3} = \frac{\log^2_3 + \log^3_3}{2\log^3_3 - 2\log^2_3} \quad \frac{\log_a^b = \frac{1}{\log_b^a}}{\log_3^a = \frac{1}{\log_a^3}}$

$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} = \frac{\frac{3+2}{6}}{\frac{3-2}{6}} = \frac{5}{1} = \boxed{\frac{10}{2}}$

$\Rightarrow \frac{t}{60} = \frac{10}{2} \Rightarrow \boxed{t = 300 \text{ min}}$ ✓

⑧ $m\left(\frac{۷}{۸}\right)^{\frac{t}{۷}} = \frac{۱}{۷}m \Rightarrow \left(\frac{۸}{۷}\right)^{\frac{t}{۷}} = ۷ \Rightarrow \left(\frac{t}{۷}\right)\log^{\frac{۸}{۷}} = \log^{۷}$

$\Rightarrow \frac{t}{۷} = \frac{\log^{۷}}{\log^{\frac{۸}{۷}}} = \frac{\log^{۷}_{۳}}{\log^{\frac{۸}{۷}}_{۳}} = \frac{\log^{۷}_{۳}}{۳\log^{۲}_{۳} - \log^{۷}_{۳}}$ $\log^{\frac{۸}{۷}}_{۳} = \log^{۸}_{۳} - \log^{۷}_{۳}$ $\log^{۲}_{۳} = \frac{۱}{\log^{۳}_{۲}}$

$= \frac{\frac{۱}{۰/۶}}{\frac{۳}{۱/۶} - \frac{۱}{۰/۶}} = \frac{\frac{۱}{۰/۶}}{\frac{۱/۸ - ۱}{(۱/۶)(۰/۶)}} = \frac{۱/۶}{۰/۸} = ۲$

$\Rightarrow \frac{t}{۷} = ۸ \Rightarrow$ **$t = ۵۶$ روز** ✓

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⑨ $\frac{۱۰۰}{۱۰۰} \times \left(\frac{۴۴}{۴۵}\right)^{t} = \frac{۱}{۳} \times \frac{۱۰۰}{۱۰۰} \Rightarrow \left(\frac{۹۶}{۱۰۰}\right)^{t} = \frac{۱}{۳}$ در هر مرحله غلظت محلول $\frac{۴۴}{۴۵}$ می‌شود.

$\Rightarrow (t)\log^{\frac{۹۶}{۱۰۰}} = \log^{\frac{۱}{۳}} \Rightarrow (t)(\log^{۹۶} - ۲) = -\log^{۳} \Rightarrow (t)(۵\log^{۲} + \log^{۳} - ۲) = -\log^{۳}$

$\log^{۵} = ۱ - \log^{۲} = ۰/۷$, $\log^{۳} = ۰/۴۸$, $\log^{۲} = ۰/۳$

$\Rightarrow (t)(۱/۵ + ۰/۴۸ - ۲) = -(۰/۴۸) \Rightarrow$ **$t = ۲۴$ روز** ✓

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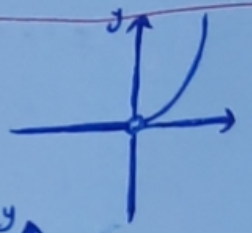
⑩

الف) $y = ۹^{\log^{x}_{۳}} = x^{\log^{۹}_{۳}} = x^{۲}$ $D_y = (۰, +\infty)$ ✓



ب) $y = \log^{x^{۲}} = ۲\log^{x}$ $x^{۲} > ۰ \Rightarrow x \neq ۰$ $D_y = \mathbb{R} - \{۰\}$ ✓